Chapter 2

NETWORK REDUCTION AND THEOREMS

2.1 INTRODUCTION

According to Kirchhoff's voltage and current laws described in the previous chapter, the algebraic sum of voltages (or that of currents) in a circuit will always remain zero. This gives rise to a voltage division in the series circuit and a current division in the parallel circuit. Source transformation and star-delta conversions are the widely used conversion techniques in the network analysis. Source transformation technique is useful in converting a practical voltage source into a current source and vice versa, whereas star-delta is used for converting one form of circuit connections to another form. Circuit analysis is simplified by systematic methods or implementation steps called network theorems. This chapter explains voltage division, current division, star-delta conversion techniques followed by network theorems in DC and AC circuits such as Thevenin's theorem, Norton's theorem, superposition theorem, maximum power transfer theorem and reciprocity theorem.

2.2 VOLTAGE DIVISION IN DC CIRCUITS

A series circuit is called a voltage divider circuit as only the voltage gets divided between the components connected in series and the current remains the same through them. According to Kirchhoff's voltage law (KVL), the sum of voltage drops across all the resistors in a series circuit is always equal to the applied voltage. This implies that the applied voltage gets proportionally divided among these resistors. The voltage division occurs across each resistor in a series circuit based on the magnitudes of individual resistances. Larger resistance will get larger portion of the applied voltage and vice versa.

Consider an example of a series circuit as shown in Fig. 2.1. This circuit consists of a voltage source V_s connected in series with three resistors R_1, R_2 and R_3. It is called a voltage divider circuit as the applied voltage V_s is proportionately divided into V_1, V_2 and V_3 among the resistors R_1, R_2 and R_3 respectively.

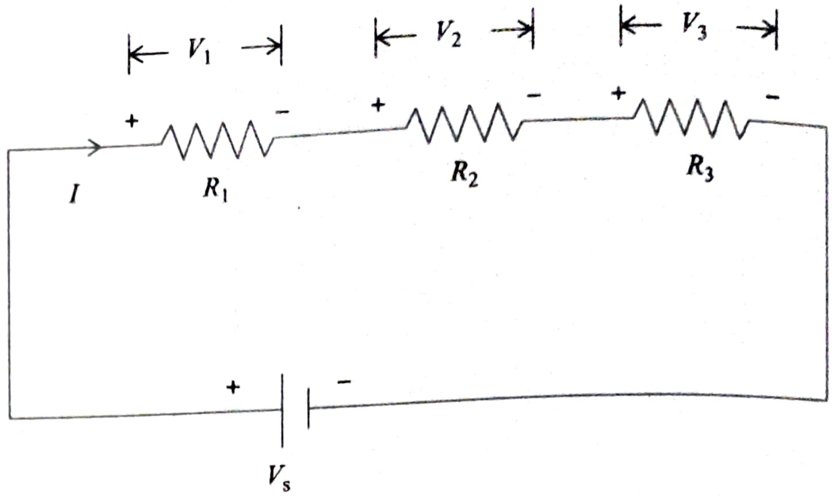

FIG. 2.1 Voltage division in a series circuit

According to Ohm's law, the voltage V_1 across resistor R_1 is

$$V_1 = IR_1$$

Similarly, $$V_2 = IR_2$$

and $$V_3 = IR_3$$

Inspection of Fig. 2.1 reveals that the current I remains the same through all these resistors and only the voltage gets divided.

Applying KVL to this circuit, the divided voltages can be summed up to the applied voltage V_s as

$$V_s = V_1 + V_2 + V_3$$

$$= IR_1 + IR_2 + IR_3$$

or $$I = \frac{V_s}{R_1 + R_2 + R_3}$$

Using the above equation, the voltage across R_1 is obtained as shown below.

$$V_1 = \frac{V_s}{R_1 + R_2 + R_3} R_1 = V_s \frac{R_1}{R_1 + R_2 + R_3}$$

Therefore, $$V_1 = V_s \frac{R_1}{R_T}, \text{ where } R_T = R_1 + R_2 + R_3.$$

Similarly, the voltages across the resistors R_2 and R_3 are

$$V_2 = IR_2 = \frac{V_s}{R_1 + R_2 + R_3} R_2 = V_s \frac{R_2}{R_T}$$

and $$V_3 = IR_3 = \frac{V_s}{R_1 + R_2 + R_3} R_3 = V_s \frac{R_3}{R_T}$$

Hence, the generic form of the voltage division can be expressed as

$$V_x = V_s \frac{R_x}{R_T} = R_x \frac{V_s}{R_T}$$

The above equation yields the voltage division rule, which states that the voltage across a resistor in a series circuit is equal to the value of that resistor times the total applied voltage divided by the total resistance of the series circuit.

Consider the series combination of two resistors as shown in Fig. 2.2(a) and its equivalent circuit shown in Fig. 2.2(b).

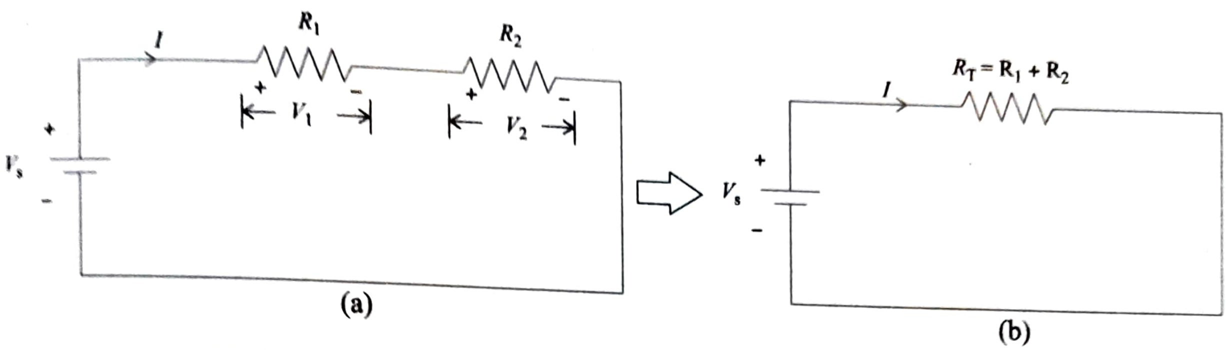

FIG. 2.2 (a) Voltage division example, and (b) its equivalent circuit

Applying KVL for the loop, we get the voltage division equation as

$$V_s = V_1 + V_2$$
$$= IR_1 + IR_2$$
$$= I(R_1 + R_2)$$
$$= IR_T \text{, where } R_T = R_1 + R_2$$

Therefore, $$I = \frac{V_s}{R_T}$$

Substituting I, V_1 is obtained as

$$V_1 = IR_1$$
$$= \frac{V_s}{R_T} R_1 = V_s \frac{R_1}{R_1 + R_2}$$

Similarly, $V_2 = IR_2$

$$= \frac{V_s}{R_T} R_2 = V_s \frac{R_2}{R_1 + R_2}$$

If all series resistances are equal, i.e., $R_1 = R_2 = R$, then

$$V_1 = V_s \frac{R}{R+R} = V_s \frac{R}{2R} = \frac{V_s}{2}$$

and $$V_2 = V_s \frac{R}{R+R} = \frac{V_s}{2}$$

If a current I flows through a series circuit consisting of n number of series resistors, R_1, R_2, R_3, ..., R_n, then by applying KVL, we get

$$V_s = IR_1 + IR_2 + IR_3 + \ldots + IR_n$$

$$= I(R_1 + R_2 + R_3 + \ldots + R_n) = IR_T, \text{ where } R_T = R_1 + R$$

According to voltage division, the voltage across nth resistor is

$$V_n = V_s \times \frac{R_n}{R_1 + R_2 + R_3 + \ldots + R_n}$$

If all these n number of resistors have equal values, i.e., $R_1 = R_2 = R_3$ the voltage across each of them is equally divided as

$$V_1 = V_2 = V_3 = \ldots V_n = \frac{V_s}{n}$$

It is to be noted that the voltage is directly proportional to the resistan series resistors of different values, the largest valued resistor will receive of voltage and vice versa.

Voltage Division in AC Circuits

The voltage division rule for AC circuits is similar to that of DC circuits resistances are replaced by impedances. For a network consisting of two and Z_2 as shown in Fig. 2.3, the voltage division rule is given by

$$\overline{V}_1 = \frac{Z_1 \overline{V}_S}{Z_1 + Z_2} \text{ and } \overline{V}_2 = \frac{Z_2 \overline{V}_S}{Z_1 + Z_2}$$

where $\overline{V}_S$ is the voltage applied to the circuit, and $\overline{V}_1$ and $\overline{V}_2$ are the voltage Z_2 respectively.

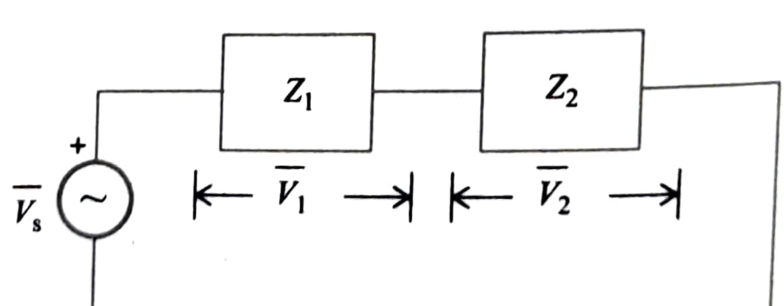

FIG. 2.3 Voltage division in an AC circuit

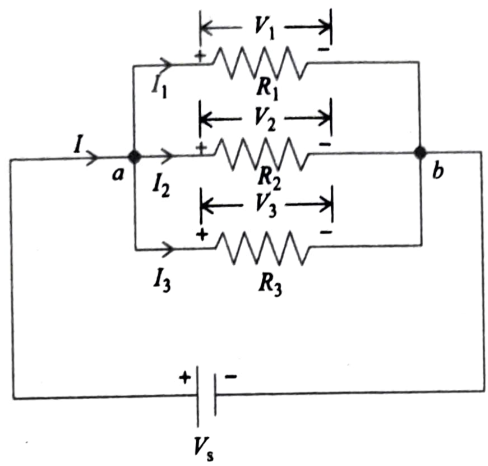

FIG. 2.4 Current division in a parallel circuit

According to Ohm's law, the current I_1 through the resistor R_1 is

$$I_1 = \frac{V_s}{R_1}$$

Similarly, $I_2 = \frac{V_s}{R_2}$ and $I_3 = \frac{V_s}{R_3}$

Inspection of Fig. 2.4 reveals that the voltages across all three resistors remain the same as V_s and only the currents get divided. Applying KCL, the divided currents can be summed up to obtain the net input current I.

i.e., $I = I_1 + I_2 + I_3$

$$= \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3}$$

$$= V_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V_s}{R_T}, \text{ where } \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \text{ (or) } R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

and $V_s = IR_T$

Since the voltage is same across all the components connected in parallel, we have

$$V_s = I_1R_1 = I_2R_2 = I_3R_3$$

In a generalized form, we have

$$V_s = I_xR_x$$

Substituting $V_s = IR_T$ in the above equation, we have

$$IR_T = I_xR_x$$

or

$$I_x = I\frac{R_T}{R_x}$$

This expression is called *current division rule*, which states that the current through any branch in a parallel circuit is equal to the total resistance of the parallel circuit divided by the resistance value of the resistor and multiplied by the total current entering the parallel circuit.

Consider the parallel combination of two resistors as shown in Fig. 2.5(a) and its equivalent circuit shown in Fig. 2.5(b).

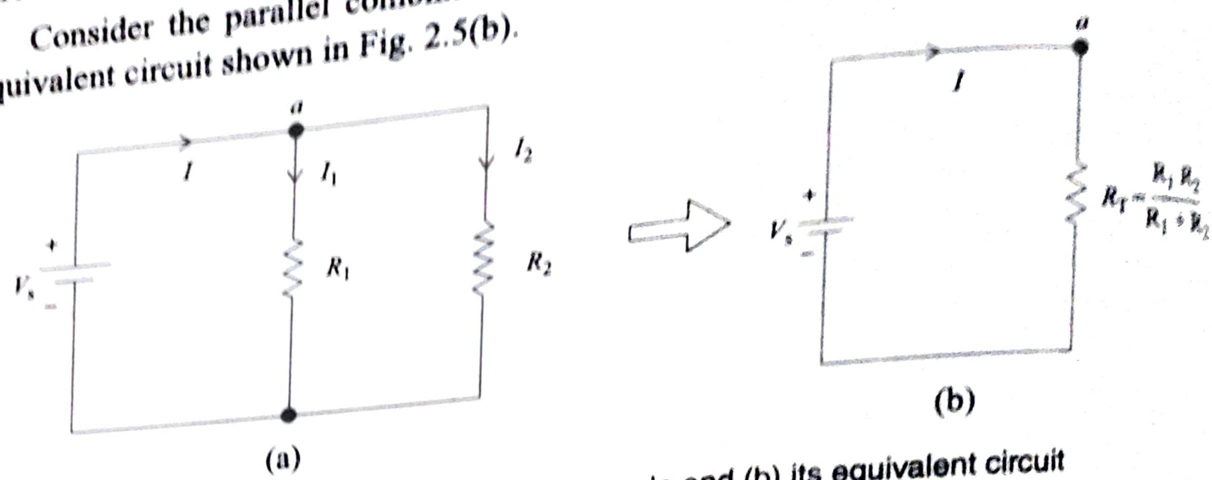

FIG. 2.5 (a) Current division example and (b) its equivalent circuit

Applying KCL at node 'a', we get the current division equation as

$$I = I_1 + I_2$$

$$= \frac{V_s}{R_1} + \frac{V_s}{R_2}$$

$$= V_s\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= \frac{V_S}{R_T}, \text{ where } \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} \quad \text{i.e., } R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Therefore, the total resistance of two parallel resistors is simply the product of their values divided by their sum.

or
$$V_s = IR_T$$

$$= I\frac{R_1 R_2}{R_1 + R_2}$$

Substituting V_s, I_1 is obtained as

$$I_1 = \frac{V_s}{R_1}$$

$$= \frac{I}{R_1}\left[\frac{R_1 R_2}{R_1 + R_2}\right] = I\frac{R_2}{R_1 + R_2}$$

Similarly,
$$I_2 = \frac{I}{R_2}\left[\frac{R_1 R_2}{R_1 + R_2}\right] = I\frac{R_1}{R_1 + R_2}$$

If all parallel resistances are equal, i.e., $R_1 = R_2 = R$, then the resistance value of the resistor and multiplied by the total current entering the parallel circuit.

Consider the parallel combination of two resistors as shown in Fig. 2.5(a) and its equivalent circuit shown in Fig. 2.5(b).

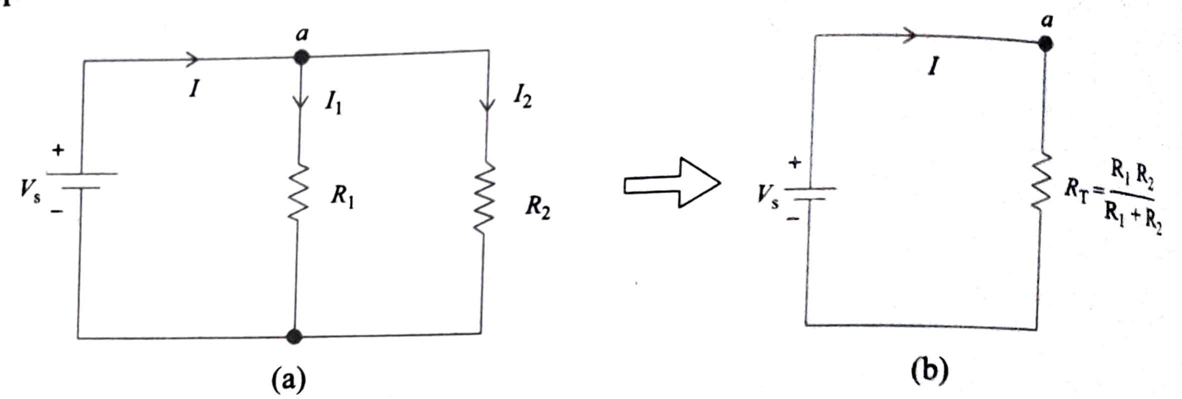

FIG. 2.5 (a) Current division example and (b) its equivalent circuit

Applying KCL at node 'a', we get the current division equation as

$$I = I_1 + I_2$$

$$= \frac{V_s}{R_1} + \frac{V_s}{R_2}$$

$$= V_s\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= \frac{V_S}{R_T}, \text{ where } \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} \quad \text{i.e., } R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Therefore, the total resistance of two parallel resistors is simply the product of their values divided by their sum.

or $$V_s = IR_T$$

$$= I\frac{R_1 R_2}{R_1 + R_2}$$

Substituting V_s, I_1 is obtained as

$$I_1 = \frac{V_s}{R_1}$$

$$= \frac{I}{R_1}\left[\frac{R_1 R_2}{R_1 + R_2}\right] = I\frac{R_2}{R_1 + R_2}$$

Similarly, $$I_2 = \frac{I}{R_2}\left[\frac{R_1 R_2}{R_1 + R_2}\right] = I\frac{R_1}{R_1 + R_2}$$

If all parallel resistances are equal, i.e., $R_1 = R_2 = R$, then

$$I_1 = I\frac{R}{R+R} = I\frac{R}{2R} = \frac{I}{2}$$

and $$I_2 = I\frac{R}{R+R} = \frac{I}{2}$$

If a circuit consists of n parallel resistors $R_1, R_2, R_3, \ldots, R_n$, then the current division equation can be written in a generalized form as

$$I = I_1 + I_2 + I_3 + \ldots + I_n$$

$$= V_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}\right) = \frac{V_s}{R_T}, \text{ where } R_T = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}.$$

or $$I = V_s\left(G_1 + G_2 + G_3 + \ldots + G_n\right), \text{ where conductance } G = \frac{1}{R}.$$

$$= V_S G_T, \text{ where } G_T = G_1 + G_2 + G_3 + \ldots + G_n.$$

According to the current division rule, the current through nth resistor is

$$I_n = I \times \frac{R_1 + R_2 + R_3 + \ldots + R_n}{R_n}$$

If all these resistors have equal values, i.e., $R_1 = R_2 = R_3 = \ldots R_n = R$, then the current through each resistor is equally divided as

$$I_1 = I_2 = I_3 = \ldots I_n = \frac{I}{n}$$

It is to be noted that the current is inversely proportional to the resistance. For multiple parallel resistors of different values, the smallest valued resistor will receive the largest share of current and vice versa.

Current Division in AC Circuits

The current division rule for AC circuits is similar to that of DC circuits except that the resistances are replaced by impedances. For a network consisting of two parallel branches with impedances Z_1 and Z_2 as shown in Fig. 2.6, the current division rule is given by

$$\overline{I}_1 = \frac{Z_2\overline{I}_T}{Z_1 + Z_2} \text{ and } \overline{I}_2 = \frac{Z_1\overline{I}_T}{Z_1 + Z_2}$$

where $\overline{I}_T$ is the total current applied to the parallel branches, and $\overline{I}_1$ and $\overline{I}_2$ are the branch currents through Z_1 and Z_2, respectively.

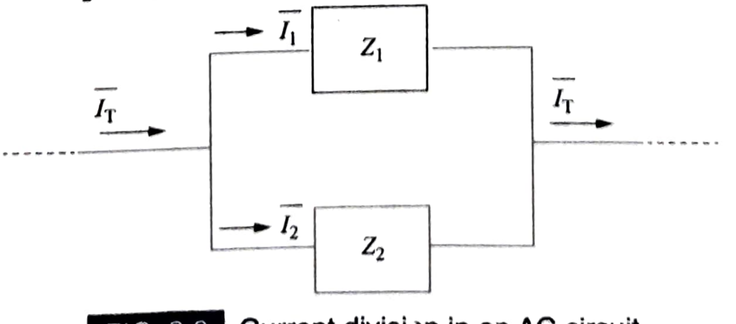

FIG. 2.6 Current division in an AC circuit