1.8 NODAL ANALYSIS METHOD

A junction point in an electrical circuit is called a node. A potential drop can be measured with respect to this point and another node acting as a reference point. Generally, grounded node is taken as a reference point. If n is the number of nodes, then the number of independent KCL equations in nodal analysis is $(n-1)$ because one node acts as a reference node.

Consider the sample circuit shown in Fig. 1.18(a).

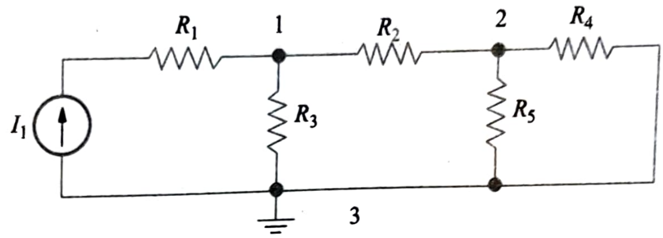

FIG. 1.18(a)

The branch currents entering and leaving node 1 can be marked as shown in Fig. 1.18(b).

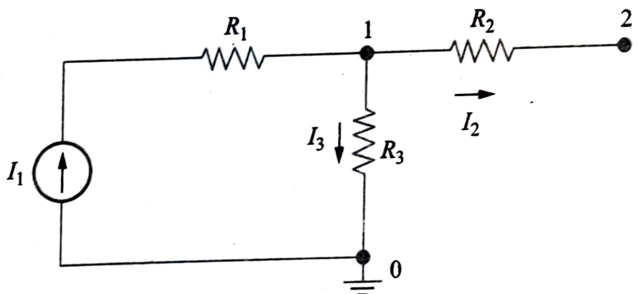

FIG. 1.18(b)

According to KCL, the currents entering the node 1 is equal to the current leaving that node. Thus,

$$I_1 = I_2 + I_3$$

According to Ohm's law, $I_2 = \dfrac{V_1 - V_2}{R_2}$ and $I_3 = \dfrac{V_1 - V_0}{R_3}$

Therefore, $\quad I_1 = \dfrac{V_1 - V_2}{R_2} + \dfrac{V_1 - V_0}{R_3}$ and since node 0 is grounded, $V_0 = 0$.

$$I_1 = \frac{V_1 - V_2}{R_2} + \frac{V_1 - 0}{R_3}$$

$$= \frac{V_1 - V_2}{R_2} + \frac{V_1}{R_3}$$

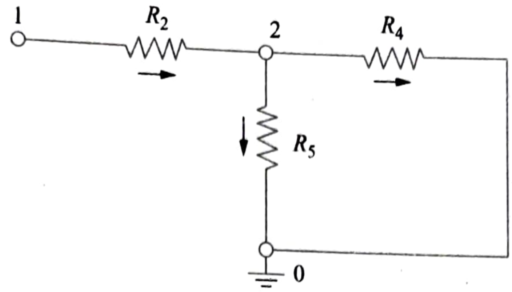

FIG. 1.18(c)

Similarly, with reference to node 2 in Fig. 1.18(c), we have

$$\frac{V_2 - V_1}{R_2} + \frac{V_2 - 0}{R_5} + \frac{V_2 - 0}{R_4} = 0$$

$$\frac{V_2 - V_1}{R_2} + V_2\left(\frac{1}{R_4} + \frac{1}{R_5}\right) = 0$$

The above nodal equations for nodes 1 and 2 are used to find the voltages at each node.

Steps involved in the nodal analysis method

Step 1: Identify all independent nodes wherever the current branches out and select a reference node.

Step 2: Write the nodal equation using KCL for all nodes except the reference node.

Step 3: Nodal equations are then solved to find nodal voltages and branch currents.

Example 1.44 Using nodal analysis, determine nodal voltages for the circuit shown in Fig. E1.44(a)

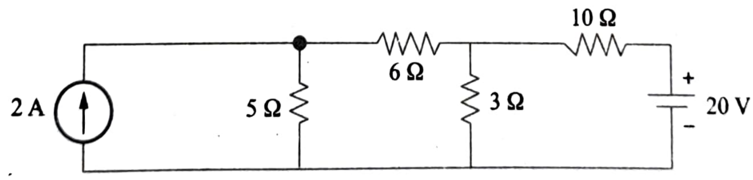

FIG. E1.44(a)

Solution

Step 1: Identify all nodes and assign a reference node as shown in Fig. E1.44(b).

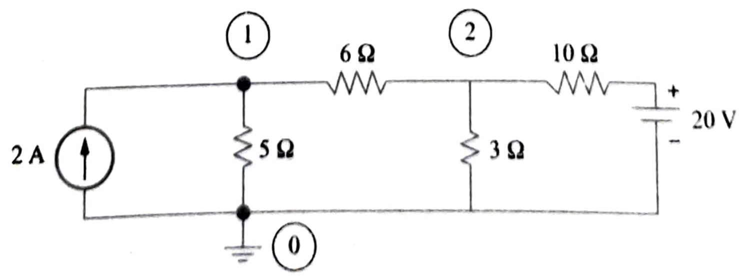

FIG. E1.44(b)

Step 2 Write the nodal equation using KCL.

At node 1,

$$2 = \frac{V_1}{5} + \frac{V_1 - V_2}{6}$$

$$V_1\left[\frac{1}{5} + \frac{1}{6}\right] - V_2\left[\frac{1}{6}\right] = 2$$

$$0.367V_1 - 0.167V_2 = 2 \quad (1)$$

At node 2,

$$\frac{V_2 - V_1}{6} + \frac{V_2}{3} + \frac{V_2 - 20}{10} = 0$$

$$\frac{-V_1}{6} + V_2\left[\frac{1}{3} + \frac{1}{6} + \frac{1}{10}\right] - 2 = 0$$

$$-V_1\left[\frac{1}{6}\right] + V_2\left[\frac{1}{3} + \frac{1}{6} + \frac{1}{10}\right] = 2$$

$$-0.167V_1 + 0.597V_2 = 2 \quad (2)$$

Step 3: Upon solving the nodal Eqn. (1) and (2), we get

$$V_1 = 7.991\,\text{V} \text{ and } V_2 = 5.585\,\text{V}$$

Example 1.45 Find the unknown voltage V_x in the circuit shown in Fig. E1.45. Assume that $V_1 = 16$ V.

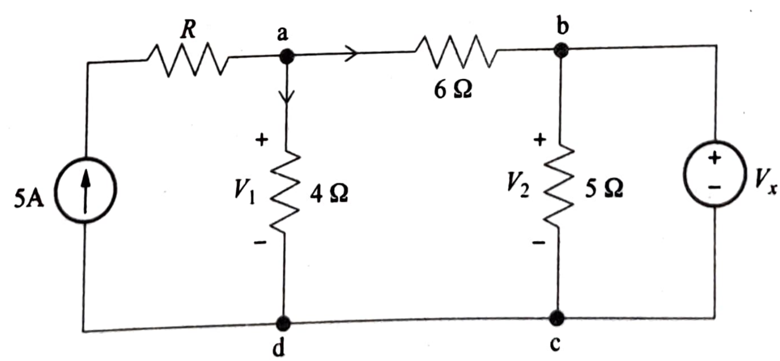

FIG. 1.45

Solution

Given $V_1 = 16$V, inspecting the circuit shown in Fig. E1.45, the current through the 4Ω resistor is $V_1/4$,

i.e., $$I_{4\Omega} = \frac{16\text{V}}{4} = 4\text{A}$$

Applying KCL at node 'a', we get

$$5 = I_{4\Omega} + I_{6\Omega} = 4 + I_{6\Omega}$$

or, $I_{6\Omega} = 5 - 4 = 1\,\text{A}$

Therefore, the voltage drop across the 6 Ω resistor, $V_{ab} = 1 \times 6 = 6\,\text{V}$

Applying KVL to the loop 'abcd', we get

$$V_2 = V_1 - V_{ab} = 16 - 6 = 10\,\text{V}.$$

Inspecting the circuit, it is noticed that $V_x = V_2$.
Hence, $V_x = 10\,\text{V}$

Example 1.46 In the circuit shown in Fig. E1.46(a), determine: (a) the open circuit voltage V_{a-b}, (b) the short-circuit current through terminals a–b, and (c) the voltage drop across 3 A current source when a–b is open-circuited.

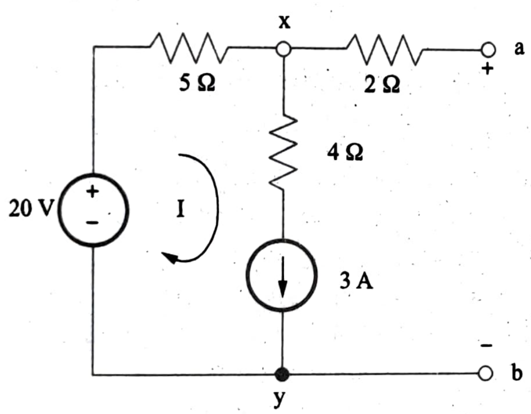

FIG. E1.46(a)

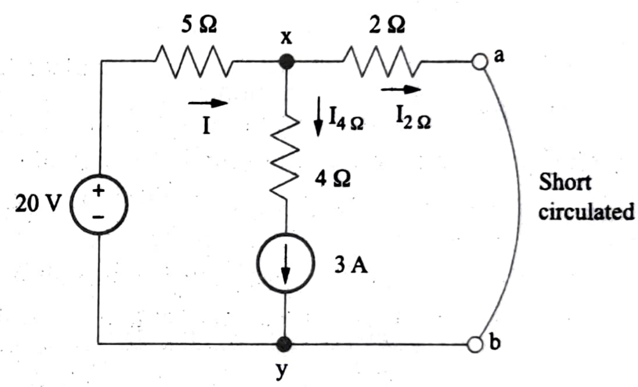

FIG. E1.46(b)

Solution

(a) Inspecting the circuit shown in Fig. E1.46(a), $I = 3\,\text{A}$

Hence, the voltage drop across 5 Ω resistor is $V_{5\Omega} = 3 \times 5 = 15\,\text{V}$

and the open-circuit voltage $V_{a-b} = V_{xy} = 20 - V_{5\Omega} = 20 - 15 = 5\,\text{V}$

(b) Short-circuiting the terminals a-b and applying KCL at node 'x', we get

$$I = I_{2\Omega} + I_{4\Omega}$$

An inspection of the circuit shown in Fig. E1.46(b) reveals that $I = \dfrac{20 - V_{xy}}{5}$, $I_{4\Omega} = 3\,\text{A}$

and since 'a' and 'b' are shorted the nodes 'a', 'b' and 'y' merge with the node 'y', so that $I_{2\Omega} = \dfrac{V_{xy}}{2}\,\text{A}$

Therefore, $\dfrac{20 - V_{xy}}{5} = 3 + \dfrac{V_{xy}}{2}$

$$\frac{20-V_{xy}}{5}=\frac{6+V_{xy}}{2}$$

or, $$V_{xy}=\frac{10}{7}\text{V}$$

Hence, the short-circuit current through a–b is

$$I_{2\Omega}=\frac{V_{xy}}{2}=\frac{10}{2\times 7}=\frac{5}{7}\text{A}$$

(c) When the a–b terminal is open-circuited as shown in Fig. E1.46(a), assuming the voltage drop across 3A source is $V_{3\Omega}$ and applying KVL to the circuit, we get

$$-20+5I+4I-V_{3\Omega}=0$$

Since $I=3$ A, $$-20+15+12-V_{3\Omega}=0$$

Therefore, $$V_{3\Omega}=7\text{ V}$$

Example 1.47 Write the nodal equations for the circuit shown in Fig. E1.47.

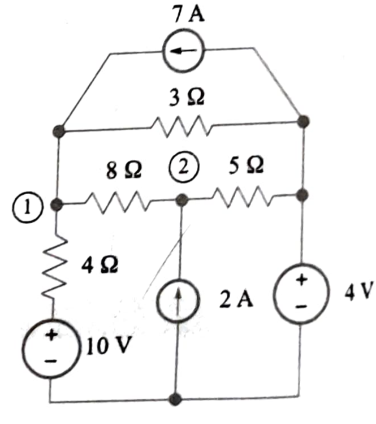

FIG. E1.47

Solution

Applying nodal analysis, we get

For node 1,

$$\frac{V_1-10}{4}+\frac{V_1-V_2}{8}+\frac{V_1-4}{3}=7$$

For node 2,

$$\frac{V_2-V_1}{8}+\frac{V_2-4}{5}=2$$

Example 1.48 For the circuit shown in Fig. E1.48, using Kirchhoff's current law, find the values of the currents, I_1 and I_2.

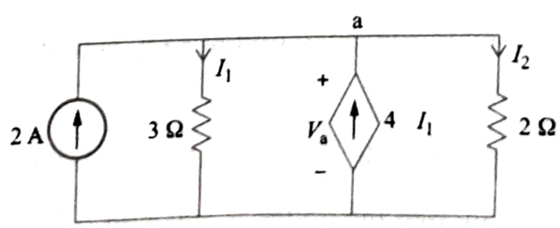

FIG. E1.48

Solution

We know that in a parallel circuit, the voltage remains the same across all the parallel branches. According to Ohm's law, $I_1=\frac{V_a}{3}$

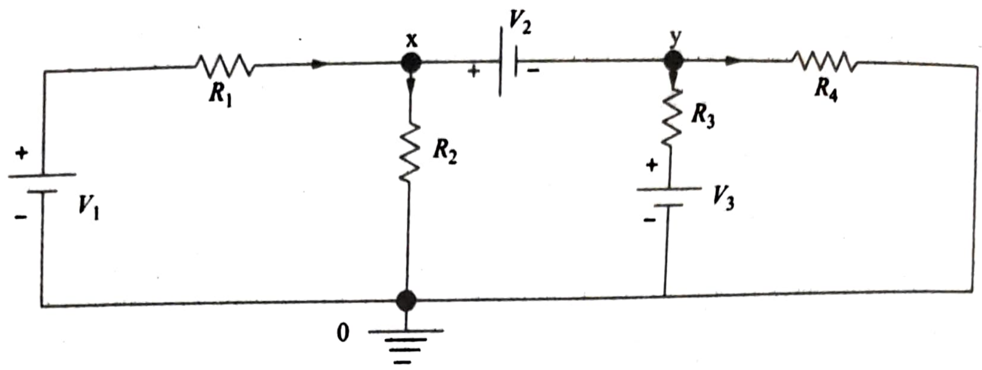

FIG. 1.19

Example 1.50 Find the current through 1 Ω resistor by using analysis method for the circuit shown in Fig. E1.50.

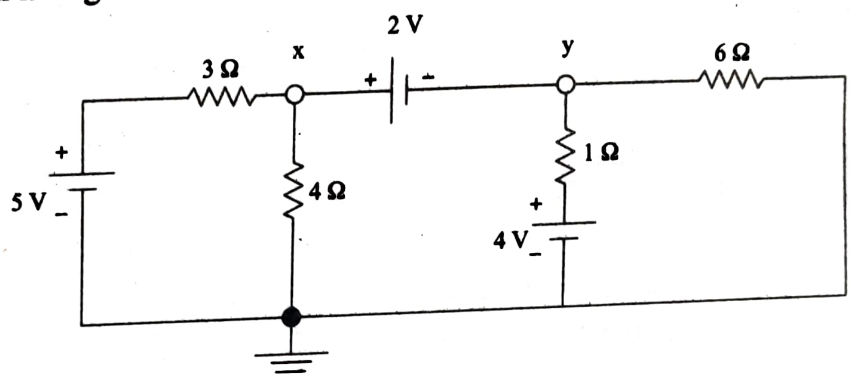

FIG. E1.50

Solution

Inspecting the circuit reveals that a voltage source is connected between two nodes 'x' and 'y'. Hence, the combined super node equation for the nodes 'x' and 'y' can be written as

$$\frac{V_x-5}{3}+\frac{V_x}{4}+\frac{V_y-4}{1}+\frac{V_y}{6}=0$$

$$V_x\left[\frac{1}{3}+\frac{1}{4}\right]+V_y\left[1+\frac{1}{6}\right]-\frac{5}{3}-4=0$$

$$0.583V_x+1.666V_y=5.666$$

We know that $V_x-V_y=2$

Solving the above equations, we get

$$V_x=4\text{V and } V_y=2\text{V}$$

The current through the 1Ω resistor, $I_{1\Omega}$

$$=\frac{V_y-4}{1}=2-4=-2\text{A}$$

Example 1.51 For the given circuit shown in Fig. E1.51, write the node voltage equations and determine the currents in each branch for the given network.

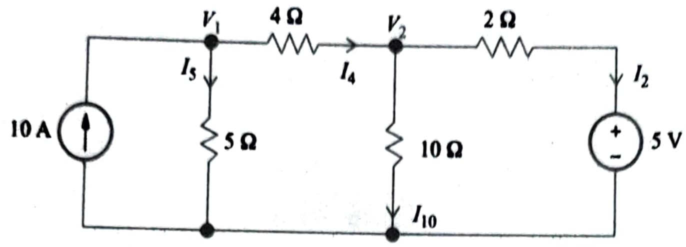

FIG. E1.51

Solution

Applying Kirchhoff's current law at node 1 we get

$$10 = \frac{V_1}{5} + \frac{V_1 - V_2}{4}$$

i.e.,
$$\frac{9}{20}V_1 - \frac{V_2}{4} = 10 \quad (1)$$

Applying KCL at node 2, we get

$$\frac{V_2 - V_1}{4} + \frac{V_2}{10} + \frac{V_2 - 5}{2} = 0$$

i.e.,
$$\frac{17}{20}V_2 - \frac{V_1}{4} = \frac{5}{2} \quad (2)$$

Solving the nodal Eqn. (1) and (2), we get

$$V_2 = 11.33\,\text{V}$$

$$V_1 = 28.5\,\text{V}$$

According to Ohm's law, the different branch currents are

$$I_5 = \frac{V_1}{5} = \frac{28.5}{5} = 5.7\,\text{A}$$

$$I_{10} = \frac{V_2}{10} = \frac{11.33}{10} = 1.133\,\text{A}$$

$$I_4 = \frac{V_1 - V_2}{4} = \frac{28.5 - 11.33}{4} = 4.2925\,\text{A}$$

and
$$I_2 = \frac{V_2 - 5}{2} = \frac{11.33 - 5}{2} = 3.165\,\text{A}$$

Example 1.52 For the given circuit shown in Fig. E1.52, determine the voltage ratio, V_o/V_i, by nodal analysis.

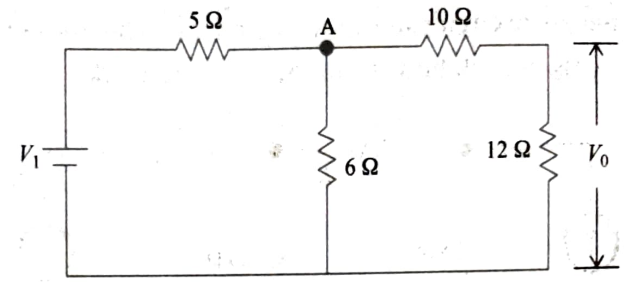

FIG. E1.52

Solution

Applying KCL at node A, we get

$$I_{5\Omega} + I_{6\Omega} + I_{22\Omega} = 0$$

i.e., $$\frac{V_A - V_I}{5} + \frac{V_A}{6} + \frac{V_A}{22} = 0$$

i.e., $$V_A\left(\frac{1}{5} + \frac{1}{6} + \frac{1}{22}\right) = \frac{V_I}{5}$$

or, $$V_A = 0.485 V_I \quad (1)$$

As the same current flows through both the resistors 10 Ω and 12 Ω, we have

$$I_{12\Omega} = I_{22\Omega}$$

i.e., $$\frac{V_A}{22} = \frac{V_o}{12}$$

$$V_A = 1.833 \times V_o \quad (2)$$

Substituting Eqn. (2) in Eqn. (1), we get

$$\frac{V_o}{V_I} = \frac{0.485}{1.833} = 0.264$$

Example 1.53 For the circuit shown in Fig. E1.53, using nodal analysis find the voltage V_i which makes the current in 10 Ω as zero.

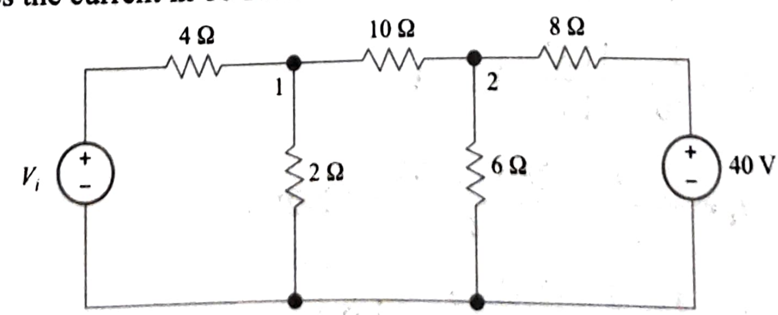

FIG. E1.53

Solution

Applying KCL at node 1, we get

$$\frac{V_1 - V}{4} + \frac{V_1}{2} + \frac{V_1 - V_2}{10} = 0$$

$$0.85V_1 - 0.1V_2 = \frac{V_i}{4} \qquad (1)$$

Applying KCL at node 2, we get

$$\frac{V_2 - V_1}{10} + \frac{V_2}{6} + \frac{V_2 - 40}{8} = 0$$

$$0.39V_2 - 0.1V_1 = 5 \qquad (2)$$

Since the current through 10Ω is zero, we have

$$V_1 - V_2 = 0$$

or, $$V_1 = V_2 \qquad (3)$$

Substituting Eqn. (3) in Eqn. (2), we get

$$V_2 = V_1 = 17.241 \text{ V}$$

Substituting these values in Eqn. (1), we get

$$V_i = 4 \times (0.85 - 0.1) \times 17.241 = 51.723 \text{ V}$$