Similarly, for the second loop,

$$(6+3)I_2 = 9$$

or,
$$I_2 = 1\text{A}$$

Voltage drop across $V_{BA} = V_{BX} + V_{XA}$

According to Ohm's law,

$$V_{BX} = -V_{6\Omega} = -6 \times I_2 = -6 \times 1 = -6\text{ V}$$

$$V_{XA} = V_{5\Omega} = 5 \times I_1 = 5 \times 1 = 5\text{V}$$

Therefore,
$$V_{BA} = -6 + 5 = -1\text{V}$$

Example 1.26 Determine the value of K_i for the circuit shown in Fig. E1.26.

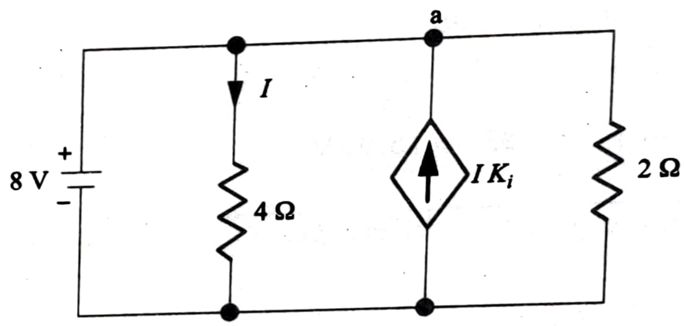

FIG. E1.26

Solution

Inspecting the circuit, it is noticed that as the resistors, 4 Ω and 2Ω, are connected in parallel with the 8 V source, the current through the 4 Ω resistor is $I = \frac{8}{4} = 2\text{A}$ and current through the 2 Ω resistor is $I_{2\Omega} = \frac{8}{2} = 4\text{A}$.

Applying KCL at node 'a', we get

$$I.K_i = I + I_{2\Omega}$$

$$2K_i = 2 + 4$$

Therefore,
$$K_i = 3.$$

1.7 MESH ANALYSIS METHOD

Kirchhoff's laws are applied in analyzing and solving the electrical circuits. The procedure of solving a complex circuit can be simplified by using either mesh or nodal analysis technique. Generally, KVL and KCL are used in deriving the mesh and nodal equations, respectively. Mesh analysis method is discussed in this section.

The term 'loop' or 'mesh' represents a closed path in the circuit through which the current can flow. Since this closed path resembles a physical fence, it is called mesh. Mesh current is the current that circulates around the mesh. If more than one meshes exist, then the current gets divided among them causing independent mesh current in each mesh. This results in independent KVL equation expressed around each of this mesh. For m number of independent meshes, totally m number of KVL equations around each mesh can be obtained.

Mesh equations are derived from the division of current in these meshes. A loop current is different from branch current. In order to illustrate this difference, consider the circuit shown in Fig. 1.16(a).

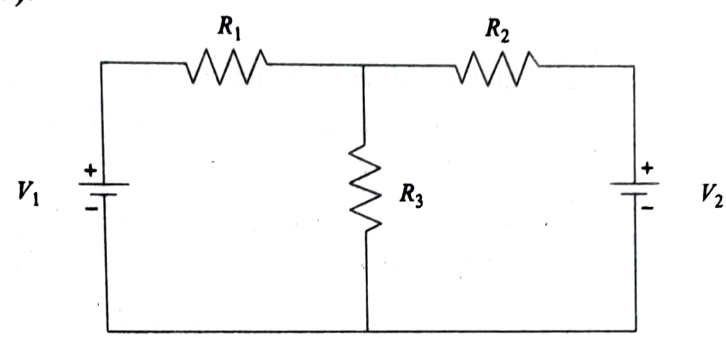

FIG. 1.16(a) A sample circuit

The given circuit is redrawn with nodes as shown in Fig. 1.16(b). Inspecting this circuit, we can identify two closed loops or meshes represented as mesh '1' (abda) and mesh '2' (bcdb). Here, I_1 and I_2 are the mesh currents flowing in mesh '1' and mesh '2', respectively.

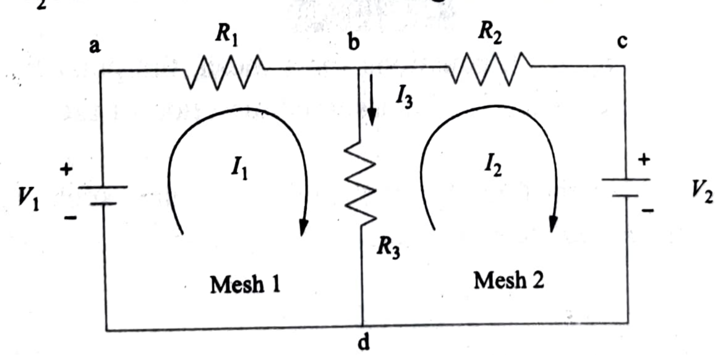

FIG. 1.16(b) Two closed loops

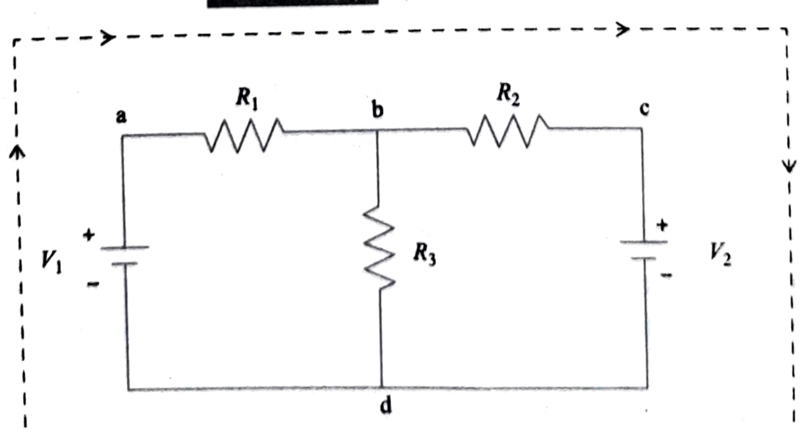

FIG. 1.16(c) Possible closed loop (abcda)

As the branch 'bd' consisting of resistor, R_3 is shared between the meshes 1 and 2, the resultant branch current is I_3. The magnitude of I_3 depends on the magnitudes of both the mesh currents, I_1 and I_2. From the circuit shown in Fig. 1.16(b), it is evident that the current directions, I_1 and I_2, in R_3 are opposite to each other. Hence,

$$I_3 = I_1 - I_2$$

Considering the circuit in Fig. 1.16(b) again, it can be noticed that there is a possibility of drawing one more closed path or mesh (abcda) through the elements $V_1 \rightarrow R_1 \rightarrow R_2 \rightarrow V_2$ and back to $\rightarrow V_1$ as shown in Fig. 1.16(c). However, as all current divisions and related voltage drops are included in the equations of first two meshes '1' and '2' itself, drawing an additional mesh is not necessary.

Assigning the direction of mesh current is arbitrary. In order to simplify the procedure for writing the mesh equation, the preferred direction for the flow of current is usually clockwise.

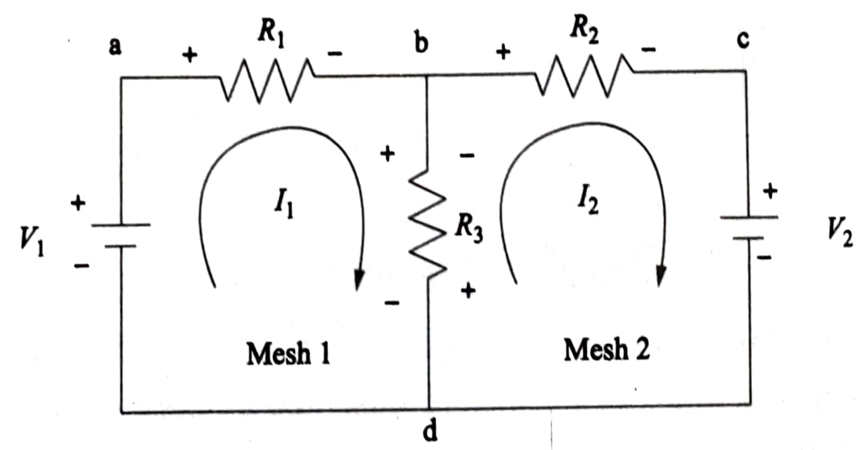

FIG. 1.16(d)

In the process of writing KVL equations for a mesh, the polarities of voltage drop across a component are determined by the assumed direction of the mesh current in that particular mesh.

If an element is located on the boundary between two meshes, such as R_3 in Fig. 1.16(d), the element current is the algebraic sum of the currents flowing through it.

Mesh Equations

Step 1: Ensure that the circuit for analysis has only voltage sources. If the circuit consists of current source, then use the source transformation techniques to convert it into a voltage source first.

Step 2: Assume the direction of current in advance and assign the identification labels.

Step 3: Along the assumed direction of current, mark the polarities of voltage drops across each element.

through common branch b-d is considered to be $(I_1 - I_2)$, whereas while writing the equation for mesh 2, the current through the same branch is considered to be $(I_1 - I_2)$.

Step 7: Solve the mesh equations to find the solution for unknown quantities.

To determine the current though I_3:

Applying KVL for mesh 1, we get the mesh equations as

$$0 = I_1R_1 + (I_1 - I_2)R_3 - V_1$$

$$V_1 = I_1(R_1 + R_3) - I_2R_3$$

For mesh 2, the mesh equations are

$$0 = I_2R_2 + V_2 + (I_2 - I_1)R_3$$

$$-V_2 = I_2(R_2 + R_3) - I_1R_3$$

These equations for V_1 and V_2 are known as mesh equations, and have to be solved further to find solution for unknown mesh currents I_1 and I_2. The branch current I_3 can be calculated from the difference between I_1 and I_2.

Mesh method is the preferred method of analysis if the circuit consists of more meshes than nodes. It is a preferred method in analyzing only the planar circuits, i.e., the circuits drawn on a plane with no crossing branches.

Example 1.27 Write mesh equations for the circuit shown in Fig. E1.27(a) and determine the currents.

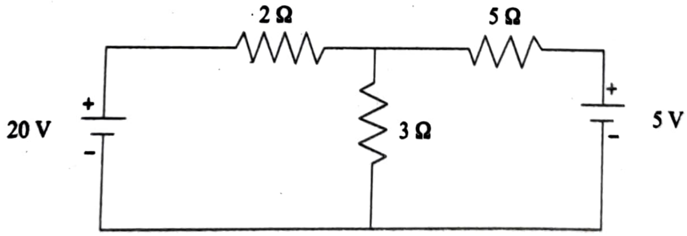

FIG. E1.27(a)

Solution

Step 1: Since the given circuit has only voltage sources there is no need of applying source transformation.

Step 2: Assume current directions and nodes and current notations as shown in Fig. E1.27(b).

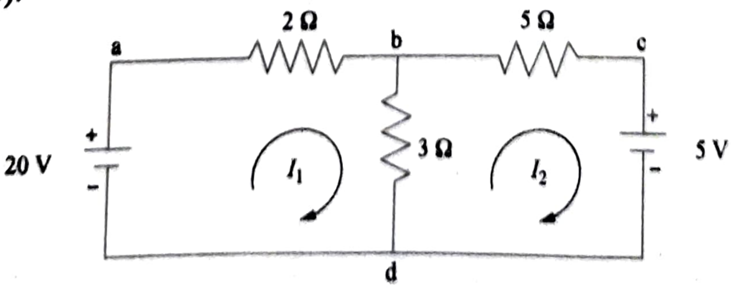

FIG. E1.27(b)

Here, we have assumed the current flow to be clockwise and current notations, I_1 and I_2, are assigned for meshes 1 (abda) and 2 (bcdb), respectively.

Step 3: Along the assumed direction of current, mark the voltage drops as +ve and −ve across each resistor as shown in Fig. E1.27(c).

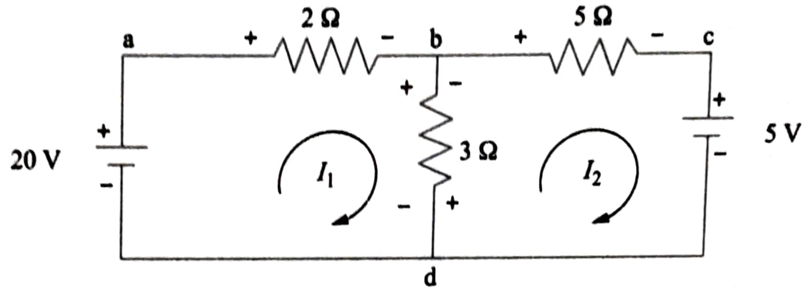

FIG. E1.27(c)

Step 4: Applying KVL, write the mesh equation for each mesh.

For mesh 1,

$$2I_1 + 3I_1 - 3I_2 = 20$$

$$2I_1 + 3(I_1 - I_2) = 20$$

$$5I_1 - 3I_2 = 20 \qquad (1)$$

For mesh 2,

$$0 = 5I_2 + 5V + 3I_2 - 3I_1$$

$$5I_2 + 3(I_2 - I_1) = -5$$

$$-3I_1 + 8I_2 = -5 \qquad (2)$$

$$(1)\times 8 \Rightarrow \ 40I_1 - 24I_2 = 160 \qquad (3)$$

$$(2)\times 3 \Rightarrow \ -9I_1 + 24I_2 = -15 \qquad (4)$$

$$(3)+(4) \Rightarrow \ 31I_1 = 145$$

Therefore, $$I_1 = \frac{145}{31} = 4.677\,\text{A}$$

Substituting the value of I_1 in Eqn. (1) and solving for I_2, we get

$$I_2 = 1.129\,\text{A}$$

Example 1.28 Determine the loop currents of the circuit shown in Fig. E1.28(a). Also, find the current through 6 Ω resistor.

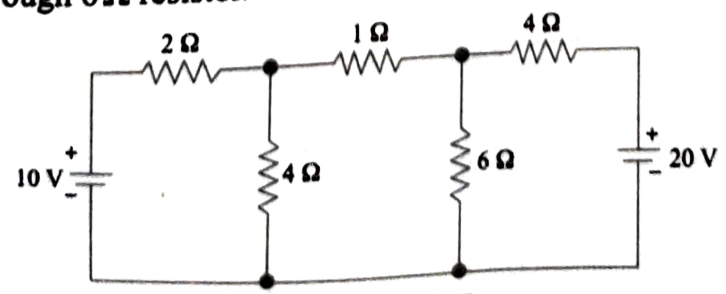

FIG. E1.28(a)

Solution

As shown in Fig. E1.28(b), the circuit consists of three meshes, 1 (abga), 2 (bcfgb) and 3 (cdefc), and the currents flowing in these meshes are I_1, I_2 and I_3, respectively.

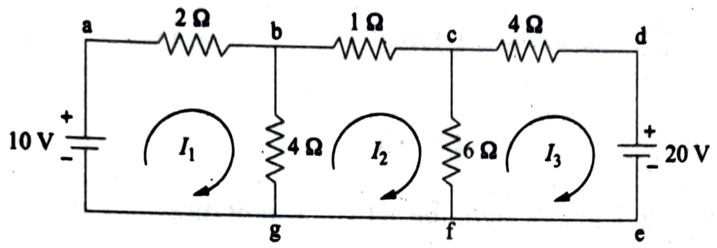

FIG. E1.28(b)

Applying KVL for mesh 1 (abga), we get

$$2I_1 + 4(I_1 - I_2) - 10 = 0$$

$$6I_1 - 4I_2 = 10 \tag{1}$$

Applying KVL for mesh 2 (bcfgb), we get

$$1.I_2 + 6(I_2 - I_3) + 4(I_2 - I_1) = 0$$

$$-4I_1 + 11I_2 - 6I_3 = 0 \tag{2}$$

Applying KVL for mesh 3 (cdefc), we get

$$4I_3 + 20 + 6(I_3 - I_2) = 0$$

$$-6I_2 + 10I_3 = -20 \tag{3}$$

Writing the mesh Eqn. (1), (2) and (3) in matrix form, we have

$$\begin{bmatrix} 6 & -4 & 0 \\ -4 & 11 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -20 \end{bmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{vmatrix} 6 & -4 & 0 \\ -4 & 11 & -6 \\ 0 & -6 & 10 \end{vmatrix} = 6(110-36) + 4(-40-0) + 0 = 444 - 160 = 284$$

$$\Delta_1 = \begin{vmatrix} 10 & -4 & 0 \\ 0 & 11 & -6 \\ -20 & -6 & 10 \end{vmatrix} = 10(110-36) + 4(0-120) + 0 = 740 - 480 = 260$$

$$\Delta_2 = \begin{vmatrix} 6 & 10 & 0 \\ -4 & 0 & -6 \\ 0 & -20 & 10 \end{vmatrix} = 6(0-120) - 10(-40+0) + 0 = -720 + 400 = -320$$

$$\Delta_3 = \begin{vmatrix} 6 & -4 & 10 \\ -4 & 11 & 0 \\ 0 & -6 & -20 \end{vmatrix} = 6(-220+0) + 4(80-0) + 10(24-0)$$

$$= -1320 + 320 + 240 = -760$$

Therefore, $$I_1 = \frac{\Delta_1}{\Delta} = \frac{260}{284} = 0.915\,\text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-320}{284} = -1.126\,\text{A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-760}{284} = -2.676\,\text{A}$$

Since I_2 and I_3 have a negative sign, the actual current direction of I_2 is opposite to the assumed clockwise direction.

Hence, $I_2 = 1.126\,\text{A}$ and $I_3 = 2.676\,\text{A}$

Current through the $6\,\Omega$ resistor is

$$I_{6\Omega} = I_3 - I_2 = 1.55\,\text{A}$$

Example 1.29 Determine the current, I_L, in the circuit shown in Fig. E1.29(a).

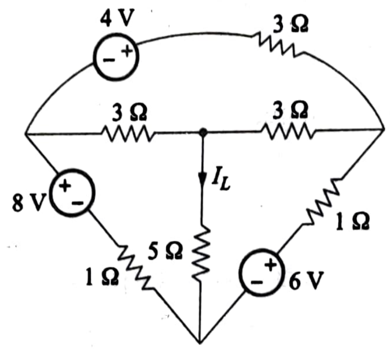

FIG. E1.29(a)

Solution

The given circuit consists of three meshes.

Assume clockwise traversal of current as shown in Fig. E1.29(b). Applying KVL, we get three mesh equations.

For mesh 1 (abca),

$$3(I_1 - I_3) + 5(I_1 - I_2) + 1I_1 - 8 = 0$$

$$9I_1 - 5I_2 - 3I_3 = 8 \quad (1)$$

For mesh 2 (bdcb),

$$3(I_2 - I_3) + 1I_2 + 6 + 5(I_2 - I_1) = 0$$

$$-5I_1 + 9I_2 - 3I_3 = -6 \quad (2)$$

For mesh 3 (adba),

$$-4 + 3I_3 + 3(I_3 - I_2) + 3(I_3 - I_1) = 0$$

$$-3I_1 - 3I_2 + 9I_3 = 4 \quad (3)$$

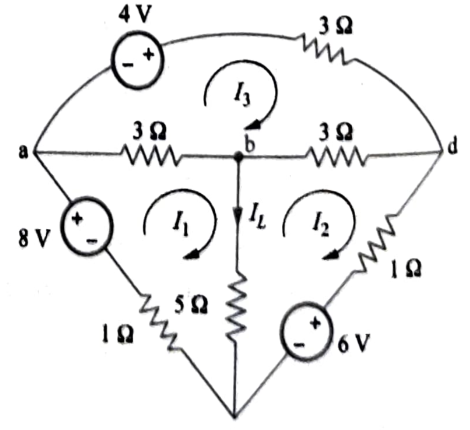

FIG. E1.29(b)

Writing mesh Eqn. (1), (2) and (3) in matrix form, we have

$$\begin{bmatrix} 9 & -5 & -3 \\ -5 & 9 & -3 \\ -3 & -3 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -6 \\ 4 \end{bmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{vmatrix} 9 & -5 & -3 \\ -5 & 9 & -3 \\ -3 & -3 & 9 \end{vmatrix} = 252$$

$$\Delta_1 = \begin{vmatrix} 8 & -5 & -3 \\ -6 & 9 & -3 \\ 4 & -3 & 9 \end{vmatrix} = 420$$

$$\Delta_2 = \begin{vmatrix} 9 & 8 & -3 \\ -5 & -6 & -3 \\ -3 & 4 & 9 \end{vmatrix} = 168$$

$$\Delta_3 = \begin{vmatrix} 9 & -5 & 8 \\ -5 & 9 & -6 \\ -3 & -3 & 4 \end{vmatrix} = 308$$

Therefore, $$I_1 = \frac{\Delta_1}{\Delta} = \frac{420}{252} = 1.667\text{ A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{168}{252} = 0.667\text{ A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{308}{252} = 1.222\text{ A}$$

Hence, $$I_L = I_1 - I_2 = 1\text{ A}.$$

Example 1.30 Determine the current through the $4\,\Omega$ resistor shown in Fig. E1.30(a).

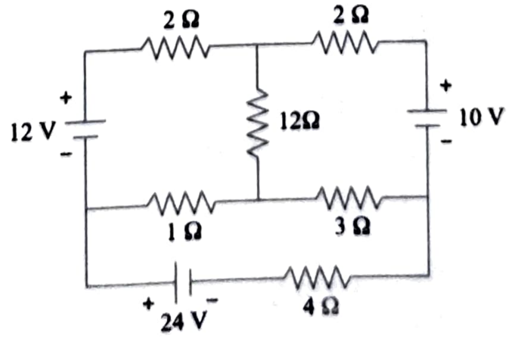

FIG. E1.30(a)

Solution

Assuming the clockwise traversal of current as shown in Fig. E1.30(b), and applying KVL to the given circuit, we get

For mesh 1,

$$2I_1 + 12(I_1 - I_2) + 1(I_1 - I_3) = 12$$

$$15I_1 - 12I_2 - I_3 = 12 \qquad (1)$$

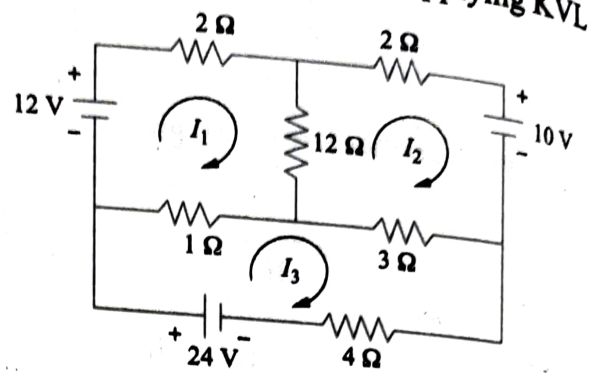

FIG. E1.30(b)

For mesh 2,

$$2I_2 + 12(I_2 - I_1) + 3(I_2 - I_3) = -10$$

$$-12I_1 + 17I_2 - 3I_3 = -10 \qquad (2)$$

For mesh 3,

$$1(I_3 - I_1) + 3(I_3 - I_2) + 4I_3 = 24$$

$$-I_1 - 3I_2 + 8I_3 = 24 \qquad (3)$$

Writing the mesh Equation. (1), (2) and (3) in matrix form, we have

$$\begin{bmatrix} 15 & -12 & -1 \\ -12 & 17 & -3 \\ -1 & -3 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -10 \\ 24 \end{bmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{vmatrix} 15 & -12 & -1 \\ -12 & 17 & -3 \\ -1 & -3 & 8 \end{vmatrix} = 664$$

$$\Delta_1 = \begin{vmatrix} 12 & -12 & -1 \\ -10 & 17 & -3 \\ 24 & -3 & 8 \end{vmatrix} = 1806$$

$$\Delta_2 = \begin{vmatrix} 15 & 12 & -1 \\ -12 & -10 & -3 \\ -1 & 24 & 8 \end{vmatrix} = 1366$$

$$\Delta_3 = \begin{vmatrix} 15 & -12 & 12 \\ -12 & 17 & -10 \\ -1 & -3 & 24 \end{vmatrix} = 2730$$

Therefore, $$I_1 = \frac{\Delta_1}{\Delta} = \frac{1806}{664} = 2.719\,\text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{1366}{664} = 2.057\,\text{A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{2730}{664} = 4.111\,\text{A}$$

The current through the 4 Ω resistor is $I_{4\Omega} = I_3 = 4.111\,\text{A}$

Example 1.31 Write mesh equations for the circuit shown in Fig. E1.31(a).

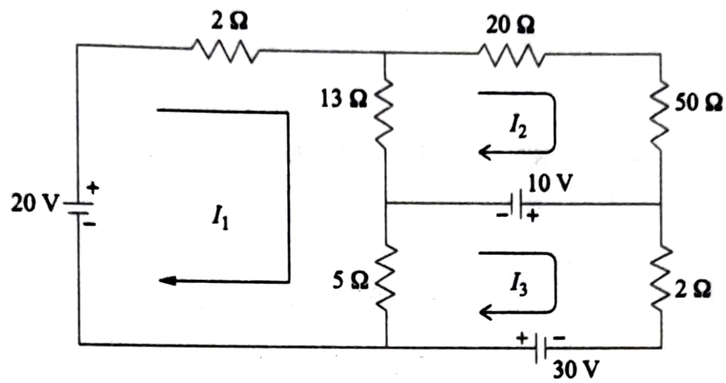

FIG. E1.31(a)

Solution

Applying KVL to this circuit, we get
For mesh 1,

$$2I_1 + 13(I_1 - I_2) + 5(I_1 - I_3) = 20$$

$$20I_1 - 13I_2 - 5I_3 = 20 \tag{1}$$

For mesh 2,

$$20I_2 + 50I_2 + 10 + 13(I_2 - I_1) = 0$$

$$-13I_1 + 83I_2 = -10 \tag{2}$$

For mesh 3,

$$-10 + 2I_3 - 30 + 5(I_3 - I_1) = 0$$

$$-5I_1 + 7I_3 = 40 \tag{3}$$

Writing the mesh Eqn. (1), (2) and (3) in matrix form, we have

$$\begin{bmatrix} 20 & -13 & -5 \\ -13 & 83 & 0 \\ -5 & 0 & 7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ -10 \\ 40 \end{bmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{vmatrix} 20 & -13 & -5 \\ -13 & 83 & 0 \\ -5 & 0 & 7 \end{vmatrix} = 8362$$

$$\Delta_1 = \begin{vmatrix} 20 & -13 & -5 \\ -10 & 83 & 0 \\ 40 & 0 & 7 \end{vmatrix} = 27310$$

$$\Delta_2 = \begin{vmatrix} 20 & 20 & -5 \\ -13 & -10 & 0 \\ -5 & 40 & 7 \end{vmatrix} = 3270$$

$$\Delta_3 = \begin{vmatrix} 20 & -13 & 20 \\ -13 & 83 & -10 \\ -5 & 0 & 40 \end{vmatrix} = 67290$$

Therefore, $$I_1 = \frac{\Delta_1}{\Delta} = \frac{27310}{8362} = 3.266\,\text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{3270}{8362} = 0.391\,\text{A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{67290}{8362} = 8.047\,\text{A}$$

Example 1.32 Determine the value of R in the circuit shown in Fig. E1.32(a), when the current is zero in the branch CD.

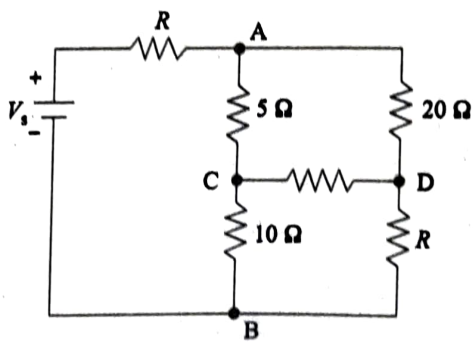

FIG. E1.32(a)

Solution

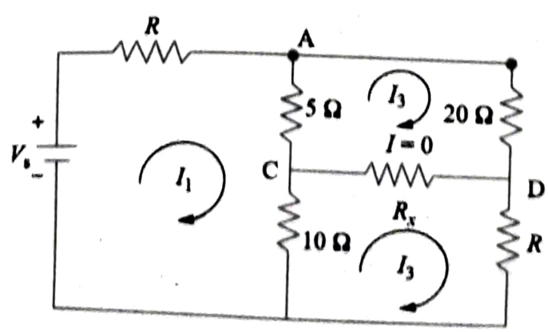

FIG. E1.32(b)

Assuming current direction to be clockwise as shown in Fig. E1.32(b), and applying KVL to mesh 1, we get

$$RI_1 + 5(I_1 - I_2) + 10(I_1 - I_3) = V_s$$

$$(15 + R)I_1 - 5I_2 - 10I_3 = V_s \tag{1}$$

Given, current in the branch CD = 0

i.e., $I_2 - I_3 = 0$

$$I_2 = I_3$$

Assuming that resistance across the CD is R_x

Applying KVL to mesh 2, we get

$$20I_2 + R_x(I_2 - I_3) + 5(I_2 - I_1) = 0$$

Substituting $I_2 - I_3 = 0$, we get

$$20I_2 + 5I_2 - 5I_1 = 0$$

$$-5I_1 + 25I_2 = 0 \tag{2}$$

Applying KVL to mesh 3, we get

$$R_x(I_3 - I_2) + RI_3 + 10(I_3 - I_1) = 0$$

Substituting $I_2 - I_3 = 0$, we get

$$-10I_1 + (10 + R)I_3 = 0 \tag{3}$$

Writing the mesh Eqn. (1), (2) and (3) in matrix form, we have

$$\begin{bmatrix} 15+R & -5 & -10 \\ -5 & 25 & 0 \\ -10 & 0 & 10+R \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \end{bmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{vmatrix} 15+R & -5 & -10 \\ -5 & 25 & 0 \\ -10 & 0 & 10+R \end{vmatrix}$$

$$= (15 + R)[25(10 + R) - 0] - 5[-5(10 + R) - 0] - 10[0 + 250]$$

$$= (15 + R)(250 + 25R) - 5(-50 - 5R) - 2500$$

$$= 3750 + 250R + 375R + 25R^2 + 250 + 25R - 2500$$

$$\Delta = 1500 + 650R + 25R^2$$

$$\Delta_1 = \begin{vmatrix} V_s & -5 & -10 \\ 0 & 25 & 0 \\ 0 & 0 & 10+R \end{vmatrix} = V_s[250 + 25R]$$

$$= 250V_s + 25V_sR$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{250V_s + 25V_sR}{1500 + 650R + 25R^2}$$

$$\Delta_2 = \begin{vmatrix} 15+R & V_s & -10 \\ -5 & 0 & 0 \\ -10 & 0 & 10+R \end{vmatrix} = (15+R)[0] - V_s[-5(10+R)-0] - 10(0)$$

$$= 50V_s + 5V_sR$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{50V_s + 5V_sR}{1500+650R+25R^2}$$

$$\Delta_3 = \begin{vmatrix} 15+R & -5 & V_s \\ -5 & 25 & 0 \\ -10 & 0 & 0 \end{vmatrix} = (15+R)(0) - 5(0) + V_s(+250) = 250V_s$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{250V_s}{1500+650R+25R^2}$$

Since $I_2 - I_3 = 0$, we have $I_2 = I_3$

$$I_3 = \frac{250V_s}{1500+650R+25R^2} = I_2 = \frac{50V_s + 5V_sR}{1500+650R+25R^2}$$

$$250V_s = 50V_s + 5V_sR$$

Therefore, $R = 40\ \Omega$

Example 1.33 Write loop and matrix equations for the network shown in Fig. E1.33 and determine the loop and branch currents in the network.

Solution

Applying KVL to the three meshes, we get

$$10I_1 - 4I_2 - 4I_3 = 4 \qquad (1)$$

$$-4I_1 + 10I_2 - 4I_3 = 0 \qquad (2)$$

$$-4I_1 - 4I_2 + 10I_3 = 0 \qquad (3)$$

Writing the mesh Eqn. (1), (2) and (3) in matrix form, we have

$$\begin{bmatrix} 10 & -4 & -4 \\ -4 & 10 & -4 \\ -4 & -4 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{vmatrix} 10 & -4 & -4 \\ -4 & 10 & -4 \\ -4 & -4 & 10 \end{vmatrix} = 392$$

$$\Delta_1 = \begin{vmatrix} 4 & -4 & -4 \\ 0 & 10 & -4 \\ 0 & -4 & 10 \end{vmatrix} = 336$$

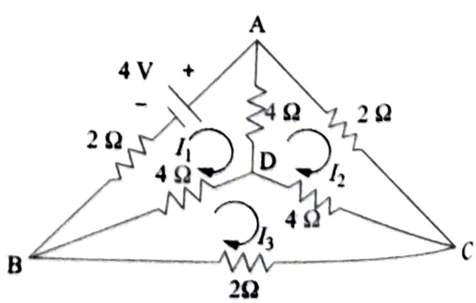

FIG. E1.33

$$\Delta_2 = \begin{vmatrix} 10 & 4 & -4 \\ -4 & 0 & -4 \\ -4 & 0 & 10 \end{vmatrix} = 224$$

$$\Delta_3 = \begin{vmatrix} 10 & -4 & 4 \\ -4 & 10 & 0 \\ -4 & -4 & 0 \end{vmatrix} = 224$$

Therefore, $$I_1 = \frac{\Delta_1}{\Delta} = \frac{336}{392} = 0.857\,\text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{224}{392} = 0.571\,\text{A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{224}{392} = 0.571\,\text{A}$$

Hence, the branch currents are

$$I_{AB} = I_1 = 0.857\,\text{A}$$

$$I_{AC} = I_2 = 0.571\,\text{A}$$

$$I_{BC} = I_3 = 0.571\,\text{A}$$

$$I_{AD} = I_1 - I_2 = 0.286\,\text{A}$$

$$I_{BD} = I_1 - I_3 = 0.286\,\text{A}$$

$$I_{CD} = I_2 - I_3 = 0$$

Example 1.34 In the circuit shown in Fig. E1.34(a), determine the current through the 2Ω resistor and the total current delivered by the battery.

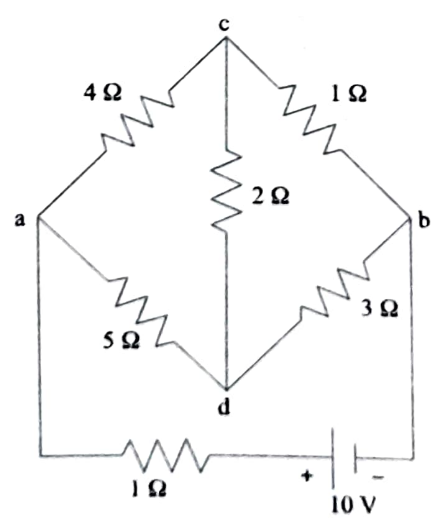

FIG. E1.34(a)

Solution

Assuming a clockwise current traversal as shown in Fig. E1.34(b) and applying KVL to the circuit, we get

$$11I_1 - 2I_2 - 5I_3 = 0 \qquad (1)$$

$$-2I_1 + 6I_2 - 3I_3 = 0 \qquad (2)$$

$$-5I_1 - 3I_2 + 9I_3 = 10 \qquad (3)$$

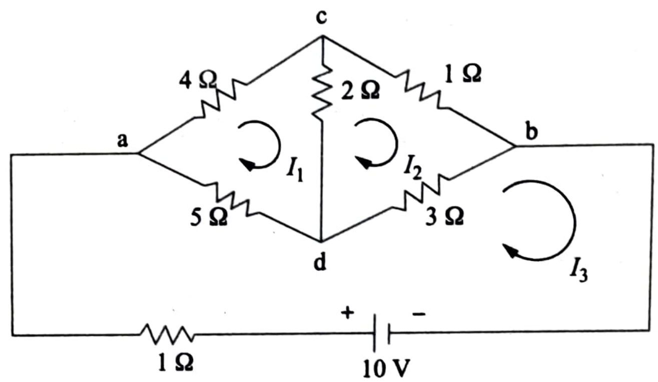

FIG. E1.34(b)

Writing the mesh Eqn. (1), (2) and (3) in matrix form, we have

$$\begin{bmatrix} 11 & -2 & -5 \\ -2 & 6 & -3 \\ -5 & -3 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{vmatrix} 11 & -2 & -5 \\ -2 & 6 & -3 \\ -5 & -3 & 9 \end{vmatrix} = 249$$

$$\Delta_1 = \begin{vmatrix} 0 & -2 & -5 \\ 0 & 6 & -3 \\ 10 & -3 & 9 \end{vmatrix} = 360$$

$$\Delta_2 = \begin{vmatrix} 11 & 0 & -5 \\ -2 & 0 & -3 \\ -5 & 10 & 9 \end{vmatrix} = 430$$

$$\Delta_3 = \begin{vmatrix} 11 & -2 & 0 \\ -2 & 6 & 0 \\ -5 & -3 & 10 \end{vmatrix} = 620$$

Therefore, $$I_1 = \frac{\Delta_1}{\Delta} = \frac{360}{249} = 1.445\,\text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{430}{249} = 1.726\,\text{A}$$

Solution

The circuit shown in Fig. E1.37(a) can be redrawn as shown in Fig. E1.37(b).

Applying KVL to the mesh 1 (acda), we get

$$R_2(I_1 - I_2) + R_1(I_1 - I_3) = V_s$$

$$(R_1 + R_2)I_1 - R_2I_2 - R_1I_3 = V_s \qquad (1)$$

Applying KVL to the mesh 2 (abca), we get

$$R_1I_2 + R_L(I_2 - I_3) + R_2(I_2 - I_1) = 0$$

$$-R_2I_1 + (R_1 + R_2 + R_L)I_2 - R_LI_3 = 0 \qquad (2)$$

Applying KVL to the mesh 3 (bdcb), we get

$$R_2I_3 + R_1(I_3 - I_1) + R_L(I_3 - I_2) = 0$$

$$-R_1I_1 - R_LI_2 + (R_2 + R_1 + R_L)I_3 = 0 \qquad (3)$$

Equations (1), (2) and (3) are the mesh equations for the given circuit.

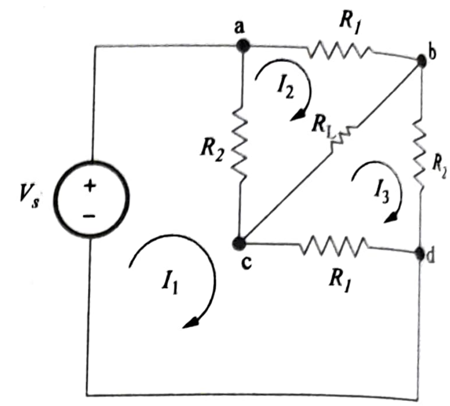

FIG. E1.37(b)

Super Mesh Analysis

The mesh equation method simplifies the analysis of the circuits consisting of only voltage sources. If the circuit has a current source in any of its branches then it will be difficult to apply mesh analysis technique directly. In that case, prior to mesh analysis the current source has to be converted into a voltage source by applying source transformation technique. Alternately, in order to simplify the analysis process, a super mesh can be formed by combining two meshes sharing a common source.

For example, consider the circuit shown in Fig. 1.17(a). Inspection of the circuit reveals three possible meshes as shown in Fig. 1.17(b). Let us assume current directions for these meshes to be clockwise.

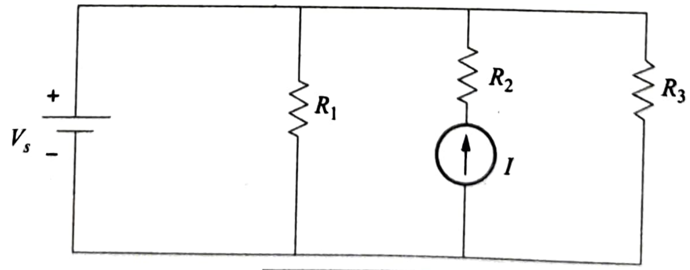

FIG. 1.17(a)

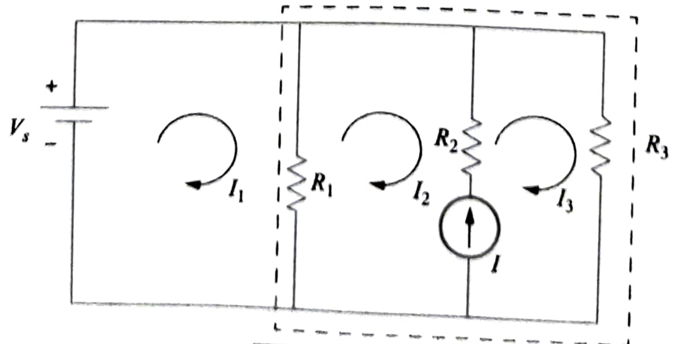

FIG. 1.17(b)

Applying KVL for the mesh 1, we get $V_s = R_1(I_1 - I_2)$. Inspecting the circuit it can be noticed that a common current source is shared between the meshes 2 and 3. A super mesh can be formed by combining these two meshes and ignoring the branch consisting of current source in the traversal path as shown by dashed lines in Fig. 1.17(b). The super mesh equation can be written as

$$R_1(I_2 - I_1) + R_3 I_3 = 0$$

The current driven by the current source, I, can be calculated as

$$I = I_3 - I_2$$

Thus, the current source shared by two meshes can be eliminated in the analysis by combining these meshes to form a super mesh.

Example 1.38 Find the mesh currents for the circuit shown in Fig. E1.38(a) by applying super mesh analysis.

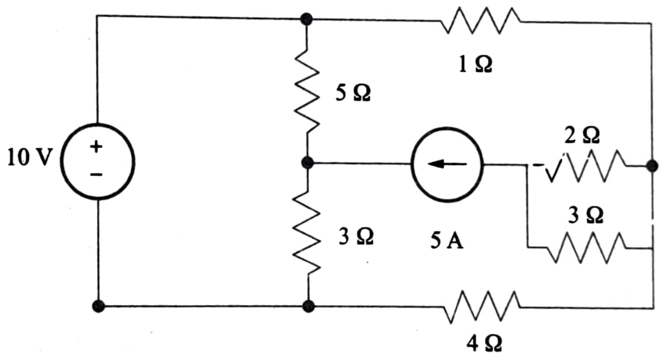

FIG. E1.38(a)

Solution

Step 1: The parallel combination of resistors 2 Ω and 3 Ω can be reduced to its equivalent value $R_{eq} = \dfrac{2\times 3}{2+3} = 1.2\,\Omega$ as shown in Fig. E1.38(b).

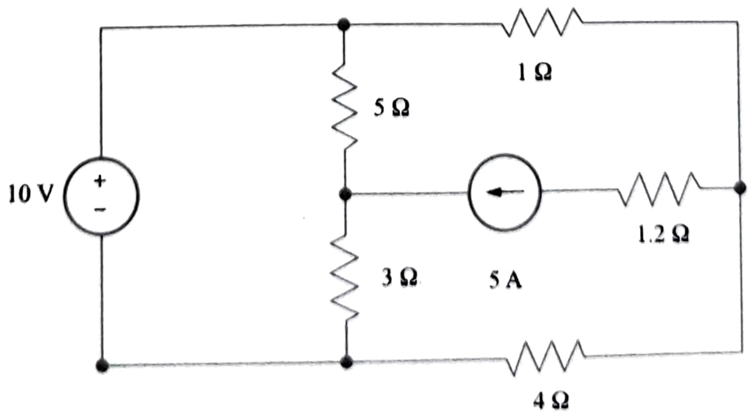

FIG. E1.38(b)

Step 2: Assume the clockwise current directions for I_1, I_2 and I_3 assign nodes as shown in Fig. E1.38(c).

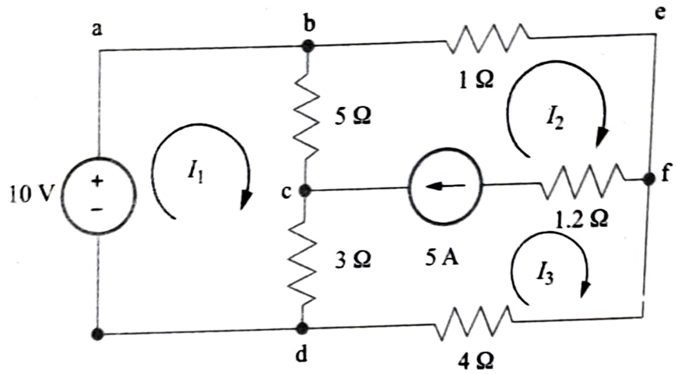

FIG. E1.38(c)

Step 3: Write the KVL equations for each mesh, and combine the meshes sharing a current source to form a super mesh.

Applying KVL for the mesh 1 (abcda), we get

$$5(I_1 - I_2) + 3(I_1 - I_3) = 10$$

$$5I_1 - 5I_2 + 3I_1 - 3I_3 = 10$$

$$8I_1 - 5I_2 - 3I_3 = 10 \tag{1}$$

As the meshes 2 (befcb) and 3 (cfdc) share a common current source of 5 A between their nodes 'c' and 'f', these two meshes are combined to form a single super mesh. The components connected in the shared branch c-f are not considered in the super mesh. Writing the KVL equation for this super mesh efdb, we get

$$5(I_2 - I_1) + 1I_2 + 4I_3 + 3(I_3 - I_1) = 0$$

$$-8I_1 + 6I_2 + 7I_3 = 0 \tag{2}$$

Adding the mesh Eqns. (1) and (2), we get

$$I_2 + 4I_3 = 10 \tag{3}$$

Since the current source of 5 A is equal to the difference between the current flowing in the meshes '2' and '3', we have

$$I_2 - I_3 = 5 \text{ or } I_2 = I_3 + 5 \tag{4}$$

Substituting Eqn. (4) in Eqn. (3), we get

$$I_3 + 5 + 4I_3 = 0$$

$$I_3 = -1\text{A}.$$

The negative sign indicates that the actual current direction is opposite to the assumed current direction.

and $\quad I_2 = I_3 + 5 = 4\text{A}$

Example 1.39 Write the mesh equations for the circuit shown in Fig. E1.39 and determine the current in each loop.

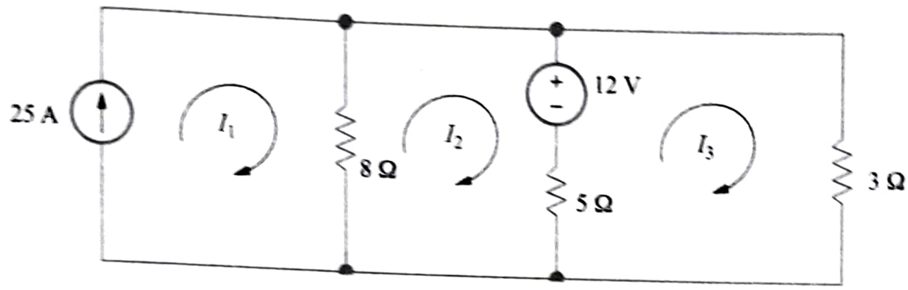

FIG. E1.39

Solution

Inspecting the circuit, mesh 1 consists of the outer perimeter current, i.e., $I_1 = 25\,\text{A}$

Applying KVL to mesh 2, we get

$$8(I_2 - I_1) + 5(I_2 - I_3) + 12 = 0$$

$$13I_2 - 5I_3 = -12 + 8 \times 25$$

$$13I_2 - 5I_3 = 188 \tag{1}$$

Applying KVL to mesh 3, we get

$$3I_3 + 5(I_3 - I_2) = 12$$

$$-5I_2 + 8I_3 = 12 \tag{2}$$

Upon solving Eqn. (1) and Eqn. (2), we get

$$I_2 = 19.797\,\text{A}, \quad I_3 = 13.875\,\text{A}$$

Example 1.40 Determine loop currents for the circuit shown in Fig. E1.40, using mesh analysis.

Solution

Inspecting the circuit, since the branches AE, DE and BC consist of current sources, applying KVL to the super mesh EBADCE, we get

$$15(I_1 - I_3) - 15 + 2I_1 + 6I_2 - 10$$

$$+ 10I_4 - 15 + 10(I_4 - I_3) = 0$$

$$17I_1 + 6I_2 - 25I_3 + 20I_4 = 40 \tag{1}$$

In the branch AE, $I_2 - I_1 = 5$ A (2)

In the branch BC, $I_3 = 20$ A (3)

In the branch DE, $I_2 - I_4 = 5$ A (4)

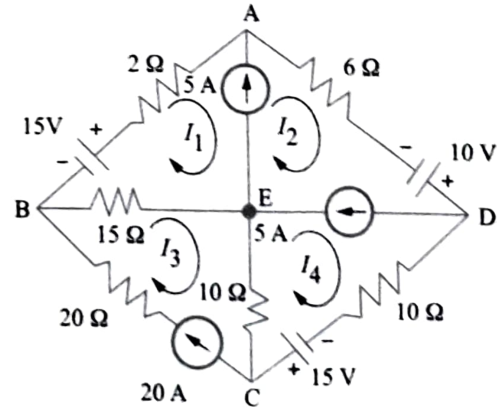

FIG. E1.40

Substituting Eqn. (2), (3), and (4) in Eqn. (1), we get

$$17I_1 + 6I_2 - 25 \times 20 + 20(-5 + I_2) = 40$$

$$17I_1 + 6I_2 - 500 - 100 + 20I_2 = 40$$

$$17I_1 + 26I_2 = 640 \tag{5}$$

Upon solving Eqn. (2) and (5), we get

$$I_1 = 11.86\,\text{A},\ I_2 = 16.86\,\text{A} \text{ and } I_4 = -5 + I_2 = 11.86\,\text{A}$$

Example 1.41 Determine the current in the $15\,\Omega$ resistor in the circuit shown in Fig. E1.41.

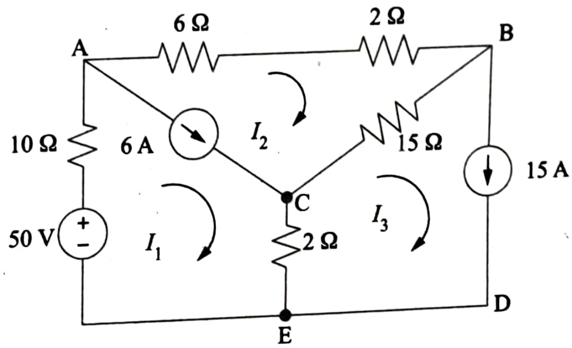

FIG E1.41

Solution

As the branches AC and BD consist of current sources, we can form a super mesh CEABC and write the equation for that as

$$2(I_1 - I_3) - 50 + 10I_1 + (6+2)I_2 + 15(I_2 - I_3) = 0$$

$$12I_1 + 23I_2 - 17I_3 = 50$$

In the branch AC, $I_1 - I_2 = 6\,\text{A}$, and in the branch BD, $I_3 = 15\,\text{A}$

Upon solving these equations, we get

$$I_1 = 12.66\,\text{A} \text{ and } I_2 = 6.66\,\text{A}$$

Example 1.42 For the given circuit shown in Fig. E1.42, find the voltage across $5\,\Omega$ resistor using mesh analysis method.

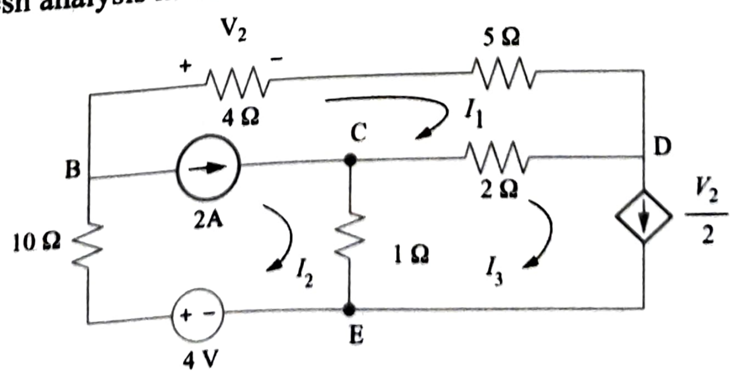

FIG. E1.42