Example 1.5 A bulb is rated as 230 V, 1 A. Find the resistance of the filament.

Solution

Given the voltage rating of the bulb, $V = 230$ V and the current rating of the bulb, $I = 1$ A.
According to Ohm's law,

$$V = IR$$

Therefore,

$$R = \frac{V}{I} = \frac{230}{1} = 230\,\Omega$$

1.4 KIRCHHOFF'S LAWS

Kirchhoff's current and voltage laws are used to systematically analyze the relationships between voltages and currents in a given electric circuit.

1.4.1 Kirchhoff's Current Law (KCL)

Kirchhoff's current law states that the algebraic sum of currents at any node is zero. That is, the total current entering a node is equal to the total current leaving that node.

In this statement, (i) the term 'node' represents a junction in a circuit, which is a common point between two or more branches. For example, Fig. 1.10 shows node x with six branches.

(ii) the term 'algebraic sum' stresses that when applying Kirchhoff's current law to a node, one has to consider not only the magnitude of the current but also its direction. In Fig. 1.10, the notations I_1, I_2, I_3, I_4, I_5 and I_6 represent magnitudes of branch currents and the arrow marks indicate their directions of flow. The individual branch current can flow towards or away from the node. From Fig. 1.10, it can be observed that three currents I_1, I_2 and I_6 are flowing towards the node x, while the other three currents I_3, I_4 and I_5 are flowing away from 'x'.

The first step in applying Kirchhoff's current law is to assign arbitrary sign conventions according to the direction of the current. Traditionally, positive sign is assigned to the currents flowing towards the node as they are treated as positive contributions to the algebraic sum, whereas the currents flowing away from the node are assigned negative sign as they are treated as negative contributions.

In Fig. 1.10, there are three positive currents I_1, I_2, and I_6 and three negative currents I_3, I_4 and I_5. Then, according to Kirchhoff's current law, the algebraic sum of the currents should be equal to zero.

$$\sum I = 0$$

or,

$$I_1 + I_2 + I_6 - I_3 - I_4 - I_5 = 0$$

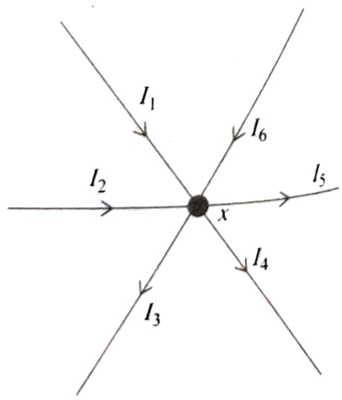

FIG. 1.10 A node x with six branches

$$(1.1)$$

Therefore, $I_1 + I_2 + I_6 = I_3 + I_4 + I_5$

If it is assumed that current entering the node has negative polarity and current leaving the node has positive polarity then the KCL can be written as

$$-I_1 - I_2 - I_6 + I_3 + I_4 + I_5 = 0$$

$$(1.2)$$

Therefore, $I_1 + I_2 + I_6 = I_3 + I_4 + I_5$

Equations (1.1) and (1.2) being the same in both the cases, it can be inferred that the total amount of current entering any node is equal to the amount of current leaving that node.

Example 1.6 Write the equation for the current at the node shown in Fig. E1.6.

Solution

The node has seven branches. The branch currents entering the node are I_1, I_2, I_5 and I_6 and those leaving the node are I_3, I_4 and I_7 Consider the current flowing towards the node as positive currents and those leaving the node as negative currents.

Applying KCL, we get

$$I_1 + I_2 + I_5 + I_6 - I_3 - I_4 - I_7 = 0$$

$$I_1 + I_2 + I_5 + I_6 = I_3 + I_4 + I_7$$

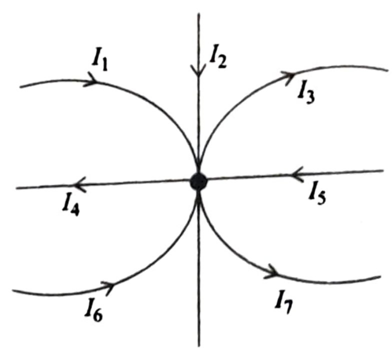

FIG. E1.6

Example 1.7 Calculate the current, I, flowing into the node shown in Fig. E1.7. Also show that the net current flowing into a node is equal to the net current flowing out of it.

Solution

Consider the current flowing towards the node as positive currents and leaving the nodes as negative currents. Applying KCL, we get

$$3 + 2 + I - 6 = 0$$

Therefore, $I = 1\text{A}$

Net current flowing into the node

$$= 3\text{A} + 2\text{A} + 1\text{A}$$

$$= 6\text{A}$$

= Net current flowing out of the node

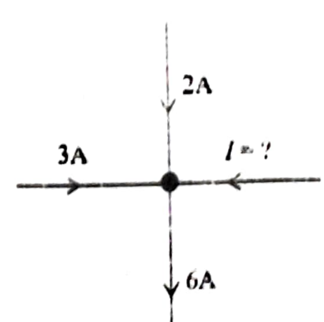

FIG. E1.7

Example 1.8 For the circuit shown in Fig. E1.8(a), calculate the value of current I.

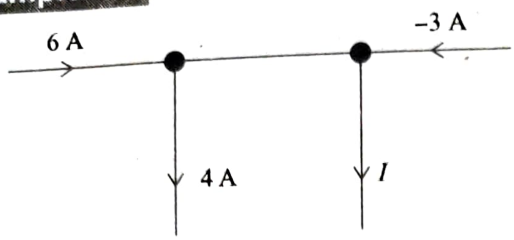

FIG. E1.8(a)

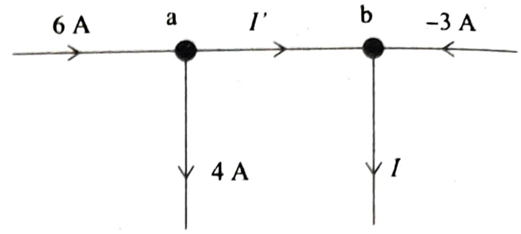

FIG. E1.8(b)

Solution

Referring to Fig. E1.8(a), there are two nodes, and each node is connected with three branches. In order to calculate I, the value of current I' flowing between two nodes 'a' and 'b' as shown in Fig. E1.8(b) has to be calculated first.

Applying KCL at node 'a', we get

$$6 = I' + 4$$

Therefore, $I' = 2$ A

Applying KCL at node 'b', we get $I' - 3 = I$. Therefore, $I = 2 - 3 = -1$ A. The negative sign in the result shows that the actual direction of the flow of current I is opposite to the direction of flow of current shown in Fig. E1.8(b).

Alternate method: Two separate nodes 'a' and 'b' can be merged into a single super node 'c', as shown in Fig. E1.8(c).

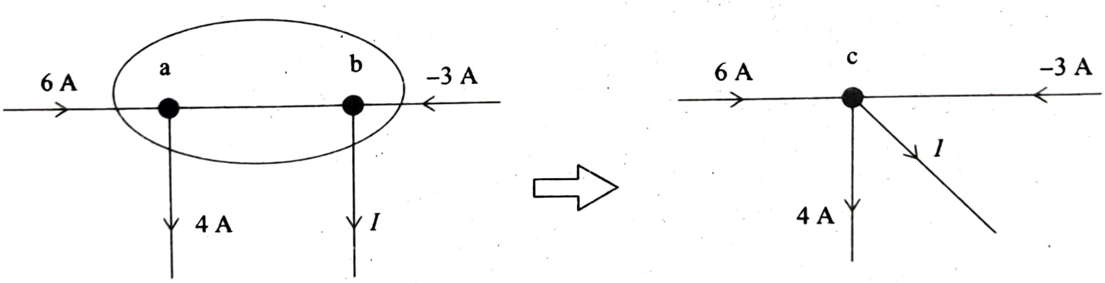

FIG. E1.8(c)

Applying KCL to super node 'c', we get $6 - 3 = 4 + I$. Therefore, $I = -1$ A. The negative sign in the result shows that the actual direction of the flow of current I is opposite to the direction of flow of current shown in Fig. E1.8(c).

1.4.2 Kirchhoff's Voltage Law (KVL)

Kirchhoff's voltage law (KVL) is the second of the Kirchhoff's laws. It states that the algebraic sum of the voltages around any closed loop or circuit is zero.

i.e., in a closed path, mesh or loop, $\sum V = 0$.

Hence, the algebraic sum of the voltage drop across the circuit elements of any closed path in a circuit is equal to the sum of the emf's or sources of voltages in that path.

i.e., $$\sum IR = \sum V$$

While applying KVL in a circuit, apart from the magnitudes of voltage drops, their directions or polarities should also be taken into account.

To illustrate the application of KVL, let us consider a single loop circuit as shown in Fig. 1.11. Here, traversal has started from node 'a' and then nodes 'b', 'c' and 'd' along a clockwise direction and ended at the same node 'a'.

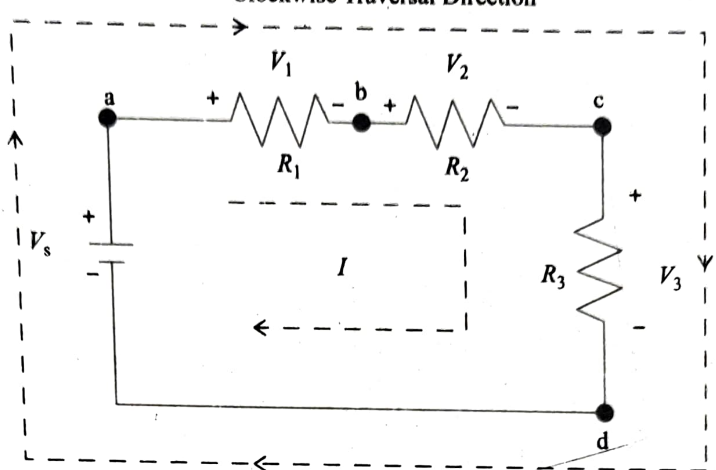

FIG. 1.11 An illustration for KVL

If I is the current flowing in the circuit, it creates a voltage drop across each resistor. Then, according to Ohm's law

$$V_1 = IR_1,\ V_2 = IR_2 \quad \text{and} \quad V_3 = IR_3$$

Here, the current direction is arbitrarily assumed and the potential drop it creates across each resistor will also have its sign convention + and –.

The current traversing the circuit in the clockwise direction as depicted by dashed lines shown in Fig. 1.11, the elements V_1 V_2 and V_3 are assigned positive + sign as its positive polarity is met first in the assumed traverse direction. Similarly, V_s is assigned a negative – sign as its negative terminal is met first in the assumed traverse direction. The starting point of the loop is entirely arbitrary but it must start and end at the same point. Thus, by applying KVL along with the traverse direction through a→b→c→d→a, we get

$$V_1 + V_2 + V_3 - V_s = 0$$

and $$\sum V = 0$$

or, $$IR_1 + IR_2 + IR_3 - V_s = 0$$

$$IR_1 + IR_2 + IR_3 = V_s$$

From the above KVL equation, the current in the circuit is calculated as

$$I(R_1 + R_2 + R_3) = V_s$$

i.e., $$I = \frac{V_s}{(R_1 + R_2 + R_3)}$$

Note: Even if the voltages in the loop are summed up in the anticlockwise direction it makes no difference except that all the signs in the resulting equation will have to be changed. Mathematically, this will end up with the same KVL equation but multiplied by –1. The result in both cases is that the algebraic sum of voltages in any loop is zero.

i.e., $\sum V = 0$.

Example 1.9 Determine the value of unknown voltage drop, V_R, for the circuit shown in Fig. E1.9.

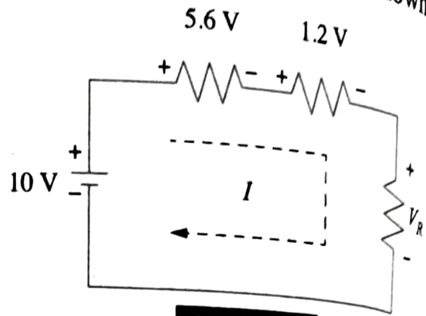

FIG. E1.9

Solution

Assuming clockwise traversal current direction and applying KVL to the given circuit, we get

$$5.6 + 1.2 + V_R - 10 = 0$$

Therefore, $\quad V_R = 3.2$ V.

Example 1.10 For the circuit shown in Fig. E1.10(a), find the circuit current and voltage across the 10 Ω resistor.

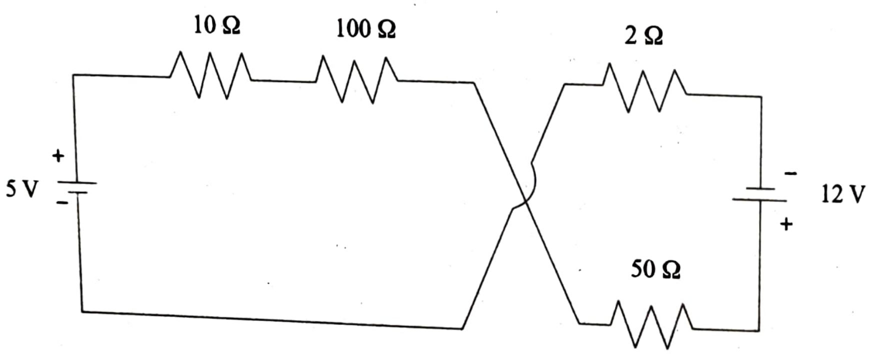

FIG. E1.10(a)

Solution

In order to simplify the analysis it is convenient to redraw the given circuit as shown in Fig. E1.10(b), in which clockwise traverse direction is assumed for the current, I, and polarities of the resistors are arbitrary.

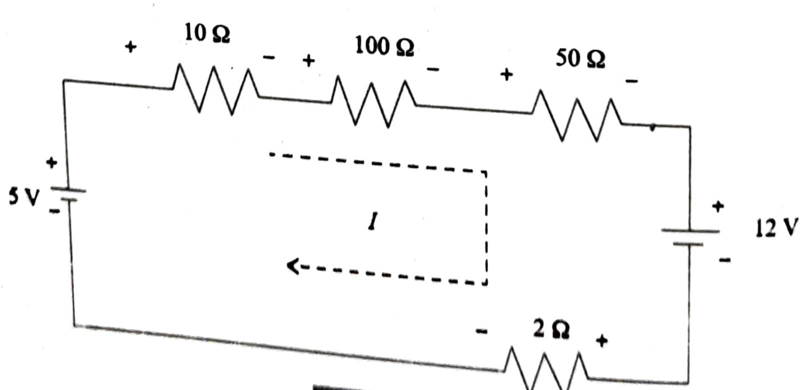

FIG. E1.10(b)

Applying KVL to the circuit, we get

$$10I + 100I + 50I + 12 + 2I - 5 = 0$$

$$162I = -7$$

Therefore, $$I = -\frac{7}{162} = -0.0432 \text{ A}$$

As the current is negative, the actual current direction is opposite to the direction of the current assumed.

Voltage across the 10Ω resistor, $V_{10\Omega} = -0.0432 \times 10 = -0.432$ V.

1.5 RESISTORS IN SERIES CIRCUITS

In a series circuit, all the components are connected in such a way that there exists only one closed path through which the current can flow. The same current flows through all the components connected in series. If a series circuit consists of only pure resistors, then it is called a series resistor circuit. The voltage across each resistor can be determined by Ohm's law.

For example, consider the series circuit shown in Fig. 1.12. It consists of resistors, R_1, R_2 and R_3 connected in series with a voltage source, V_s, across them. Due to this voltage source, a current, I, flows through the circuit.

The total, or equivalent, resistance of the circuit can be expressed as

$$R_{eq} = R_1 + R_2 + R_3$$

Applying KVL to the circuit, we get

$$V_s = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3) = IR_{eq}$$

Therefore, $$I = \frac{V_s}{R_{eq}}$$

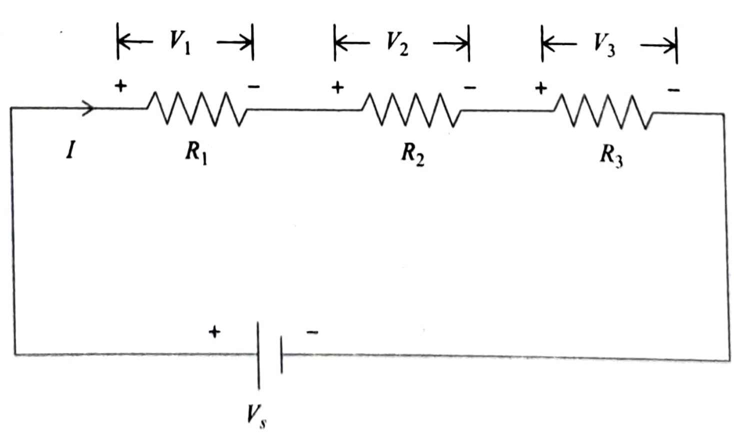

FIG. 1.12 Resistors in series

Thus, the current in the series circuit is determined by the voltage and its equivalent resistance.

The equivalent resistance in a series circuit is equal to the sum of all resistors connected in series. Suppose if n number of fixed resistors are connected in series, then

$$R_{eq} = R_1 + R_2 + R_3 + \dots\dots\dots + R_n.$$

When all resistors have equal values, then $R_{eq} = nR$.