



# SNS COLLEGE OF ENGINEERING

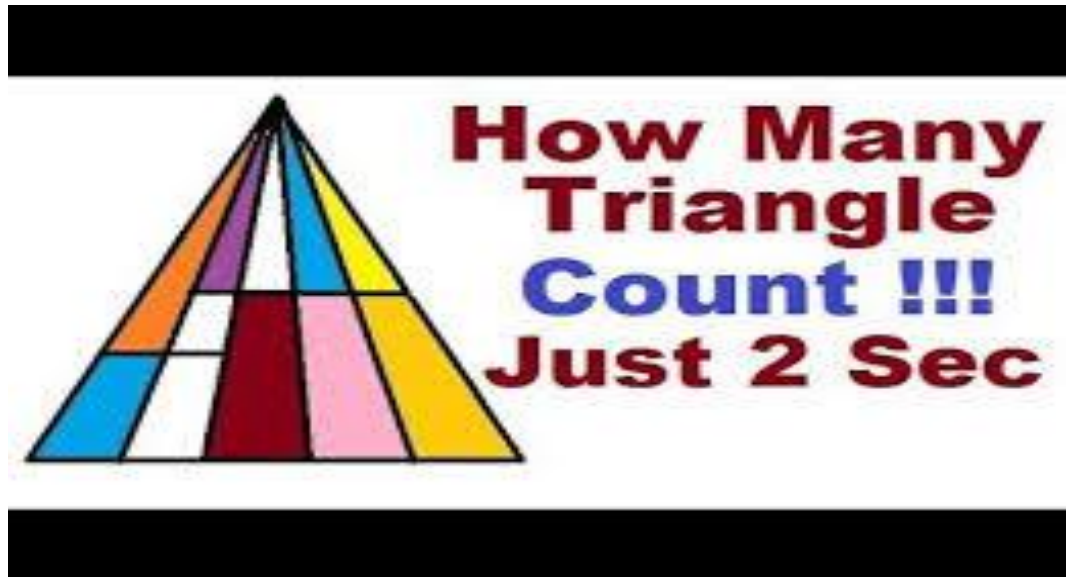
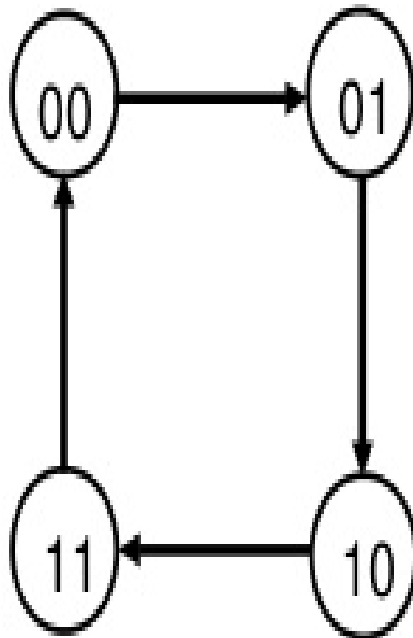
(Autonomous)

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING



## DIGITAL PRINCIPLES AND SYSTEM DESIGN

Guess Today's Topic???





## Counters

- Introduction: Counters
- Asynchronous (Ripple) Counters
- Asynchronous Counters with MOD number  $< 2^n$
- Asynchronous Down Counters
- Cascading Asynchronous Counters



- Synchronous (Parallel) Counters
- Up/Down Synchronous Counters
- Designing Synchronous Counters
- Decoding A Counter
- Counters with Parallel Load



# Introduction: Counters



- **Counters** are circuits that cycle through a specified number of states.
- Two types of counters:
  - ❖ synchronous (parallel) counters
  - ❖ asynchronous (ripple) counters
- Ripple counters allow some flip-flop outputs to be used as a source of clock for other flip-flops.
- Synchronous counters apply the same clock to all flip-flops.

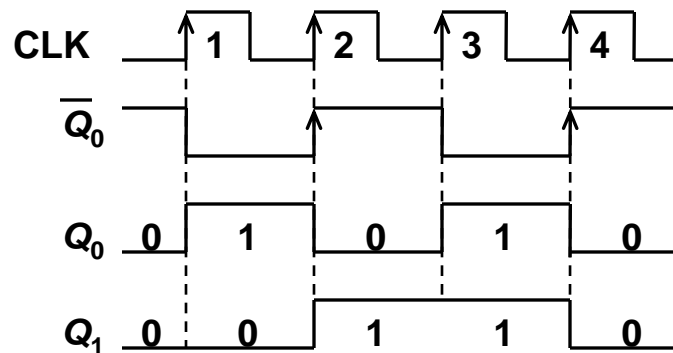
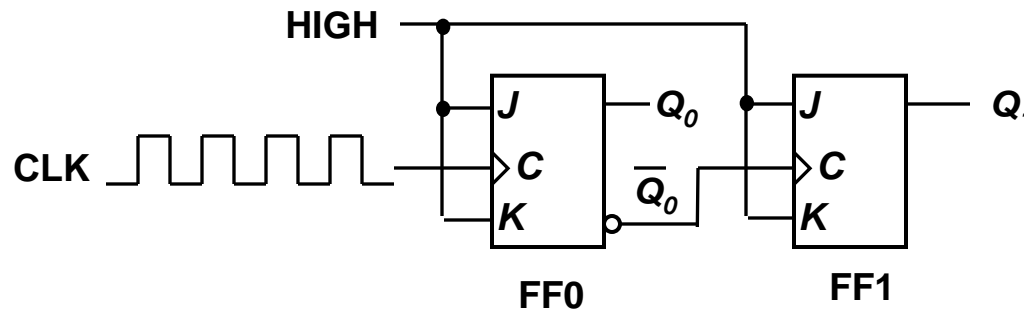


# Asynchronous (Ripple) Counters



- **Asynchronous counters**: the flip-flops do not change states at exactly the same time as they do not have a common clock pulse.
- Also known as **ripple counters**, as the input clock pulse “ripples” through the counter – cumulative delay is a drawback.
- $n$  flip-flops  $\rightarrow$  a MOD (modulus)  $2^n$  counter. (Note: A MOD- $x$  counter cycles through  $x$  states.)
- Output of the last flip-flop (MSB) divides the input clock frequency by the MOD number of the counter, hence a counter is also a *frequency divider*.

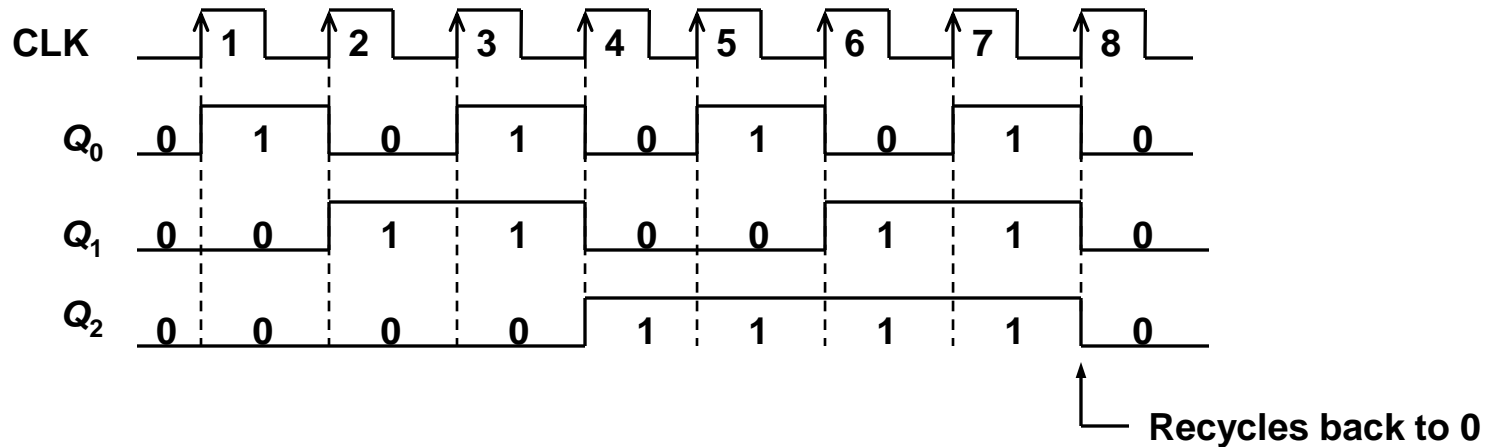
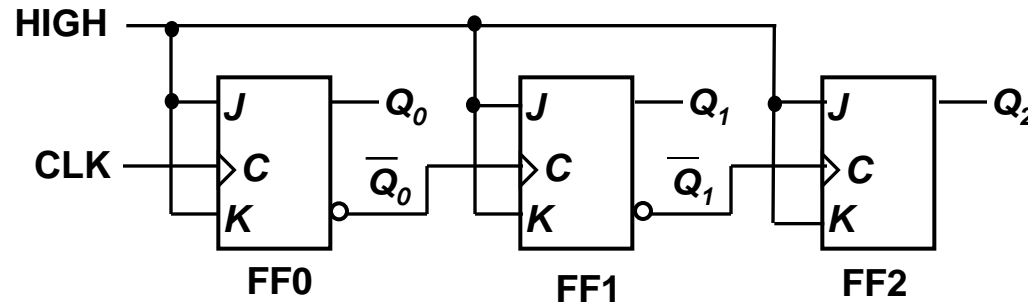
- Example: 2-bit ripple binary counter.
- Output of one flip-flop is connected to the clock in of the next more-significant flip-flop.



Timing diagram

00 → 01 → 10 → 11 → 00 ...

- Example: 3-bit ripple binary counter.

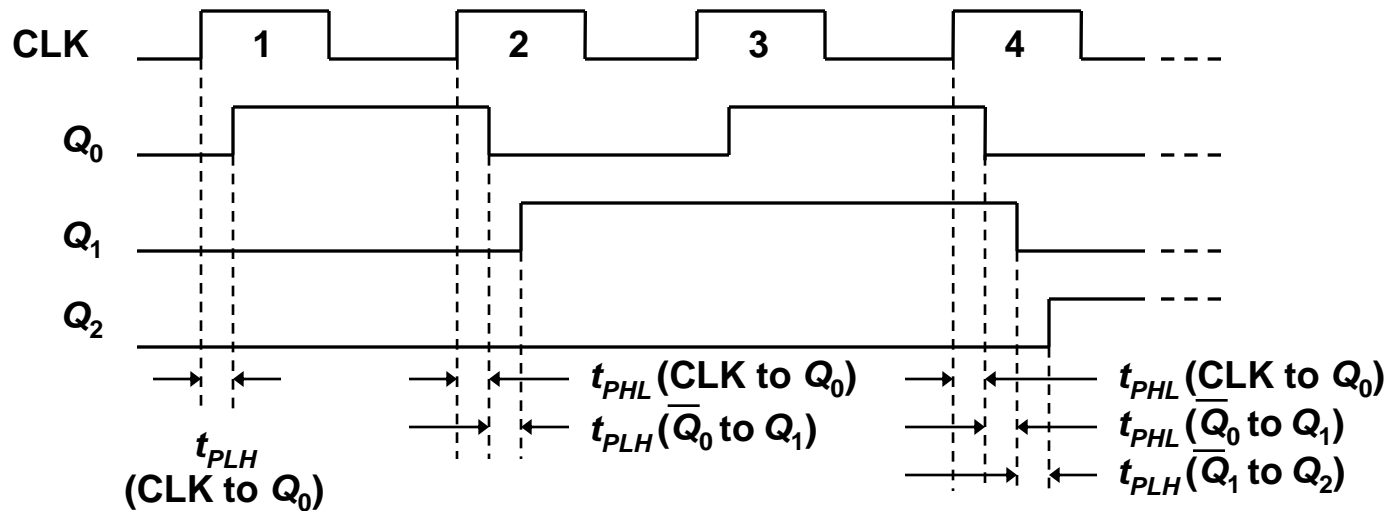




# Asynchronous (Ripple) Counters

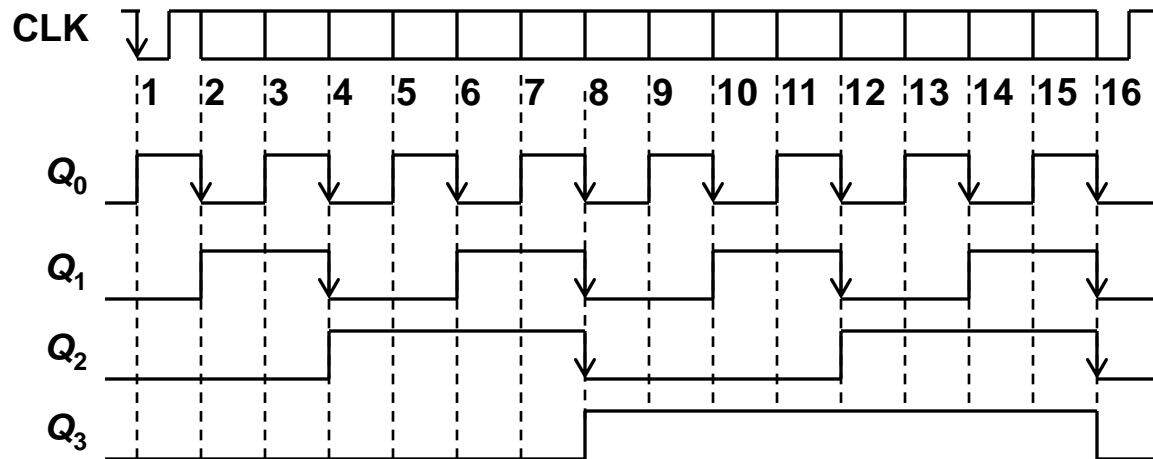
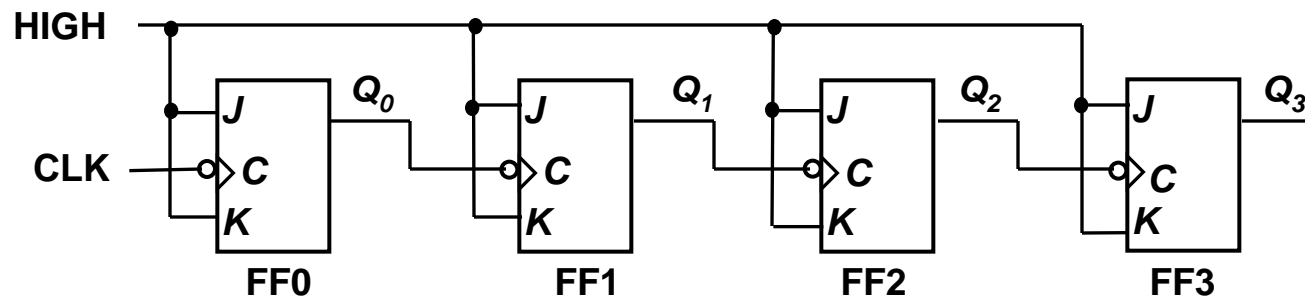


- Propagation delays in an asynchronous (ripple-clocked) binary counter.
- If the accumulated delay is greater than the clock pulse, some counter states may be misrepresented!



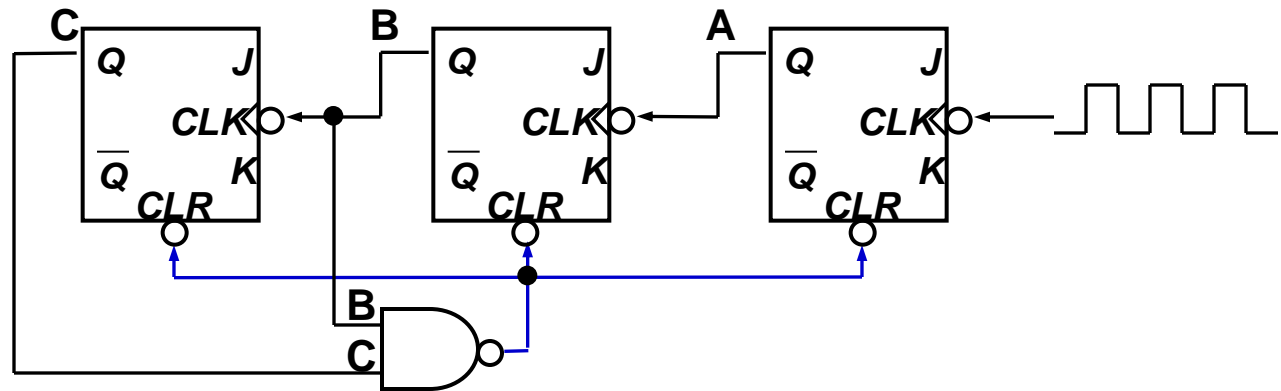


- Example: 4-bit ripple binary counter (negative-edge triggered).

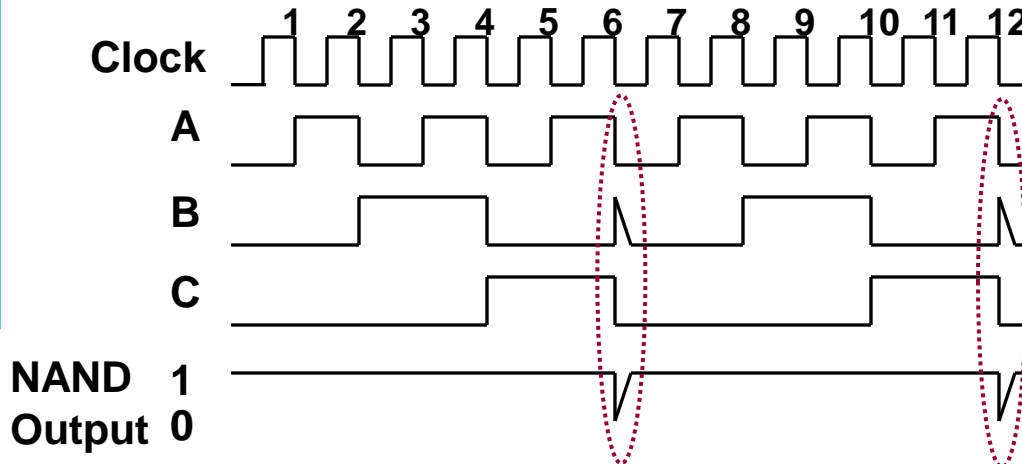
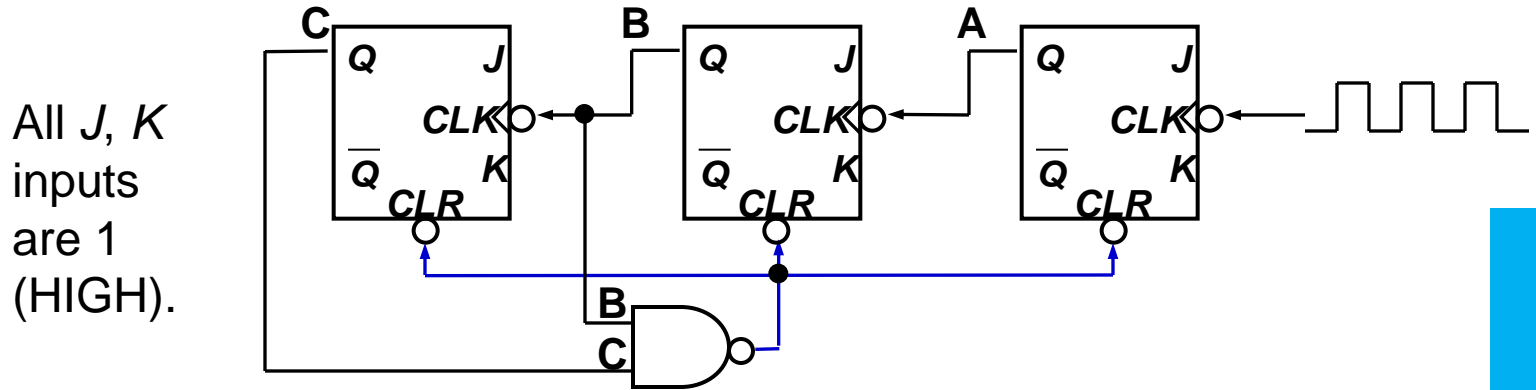


- States may be skipped resulting in a **truncated sequence**.
- Technique: force counter to *recycle before going through all of the states* in the binary sequence.
- Example: Given the following circuit, determine the counting sequence (and hence the modulus no.)

All J, K inputs are 1 (HIGH).



- Example (cont'd):



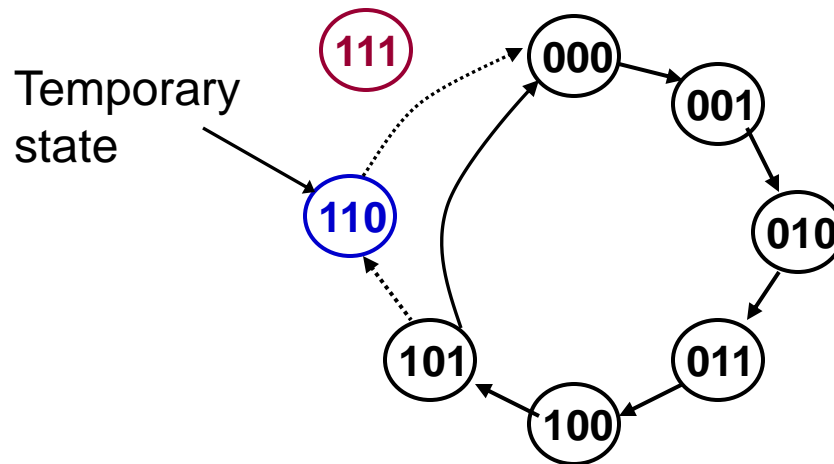
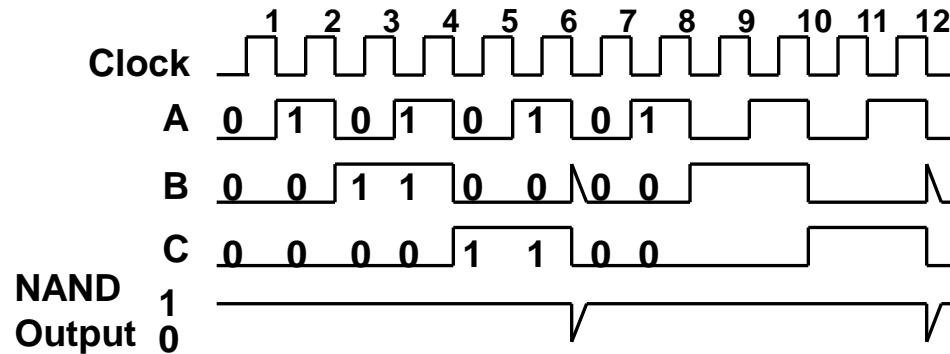
**MOD-6 counter** produced by clearing (a MOD-8 binary counter) when count of six (110) occurs.



# Asyn. Counters with MOD no. $< 2^n$

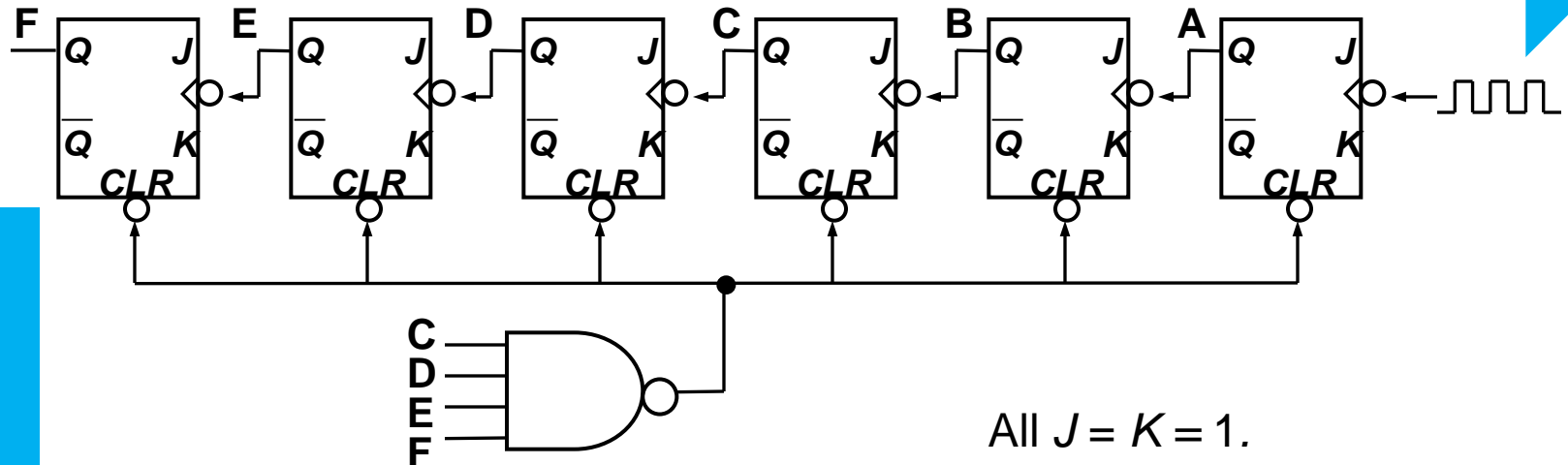


- Example (cont'd): Counting sequence of circuit (in CBA order).



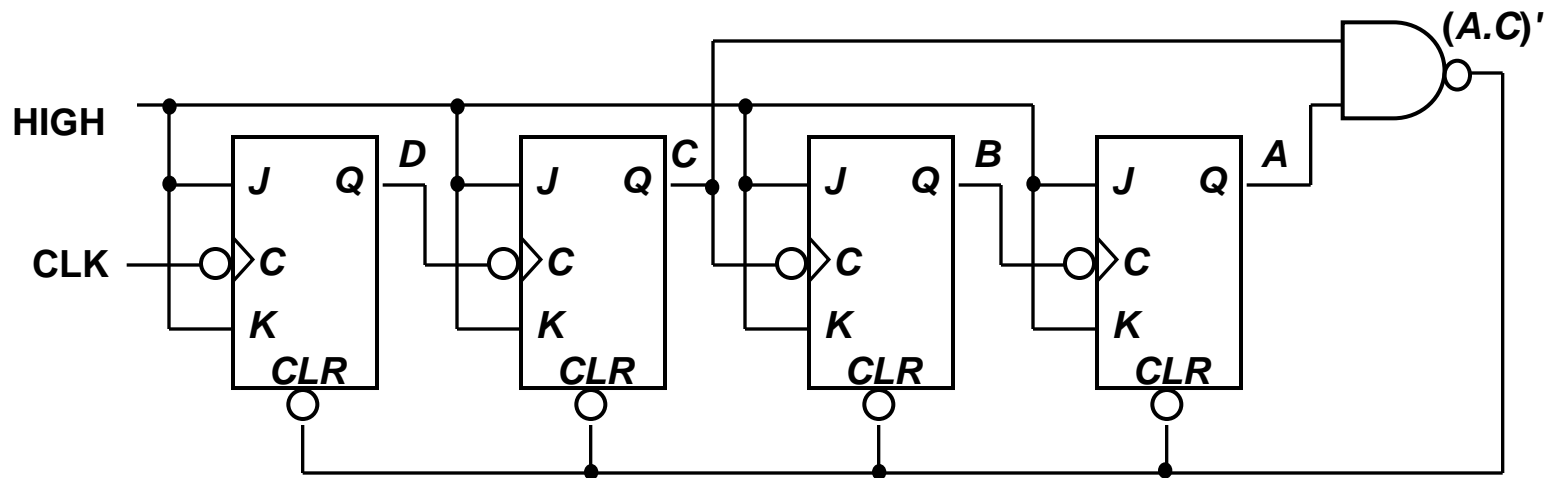
Counter is a MOD-6 counter.

- *Exercise:* How to construct an asynchronous MOD-5 counter? MOD-7 counter? MOD-12 counter?
- *Question:* The following is a MOD-? counter?

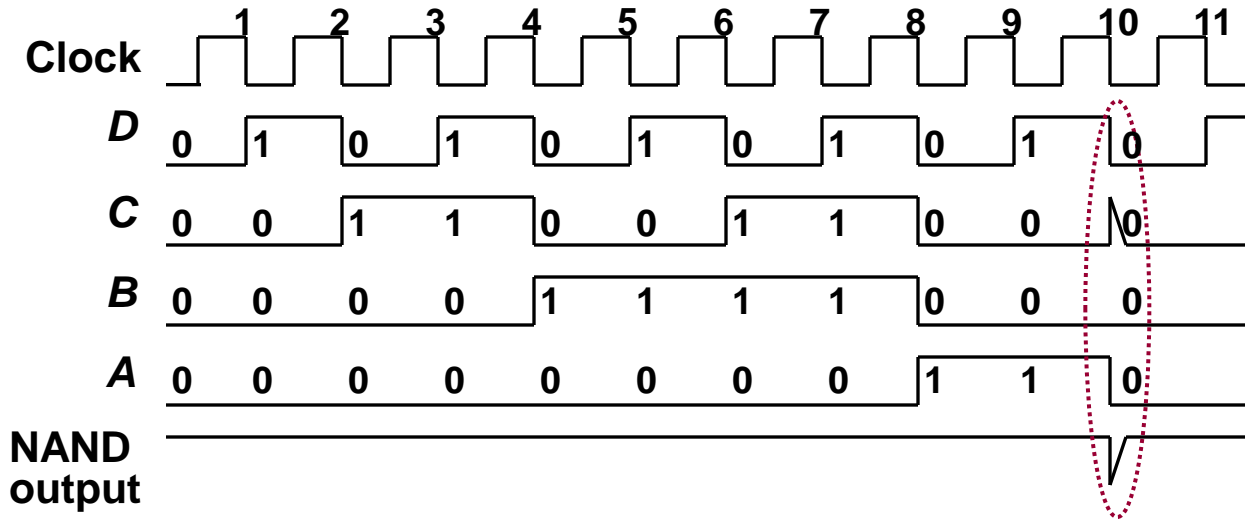
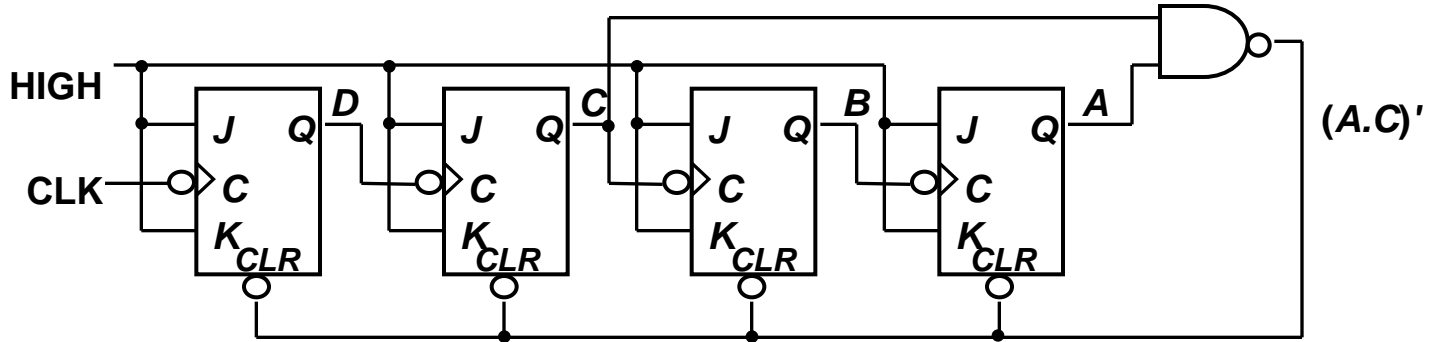


# Asyn. Counters with MOD no. $< 2^n$

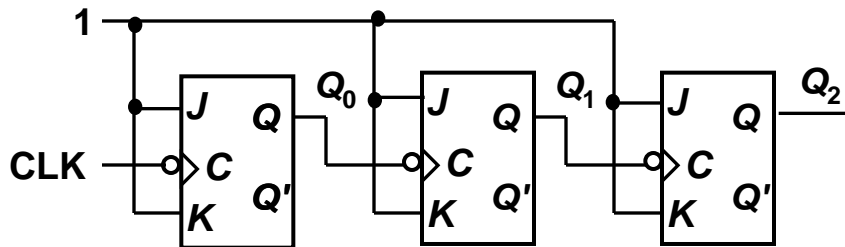
- Decade counters (or BCD counters) are counters with 10 states (modulus-10) in their sequence. They are commonly used in daily life (e.g.: utility meters, odometers, etc.).
- Design an asynchronous decade counter.



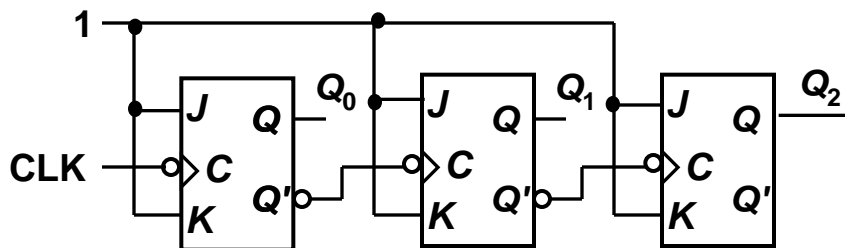
- Asynchronous decade/BCD counter (cont'd).



- So far we are dealing with *up counters*. *Down counters*, on the other hand, count downward from a maximum value to zero, and repeat.
- Example: A 3-bit binary (MOD- $2^3$ ) down counter.



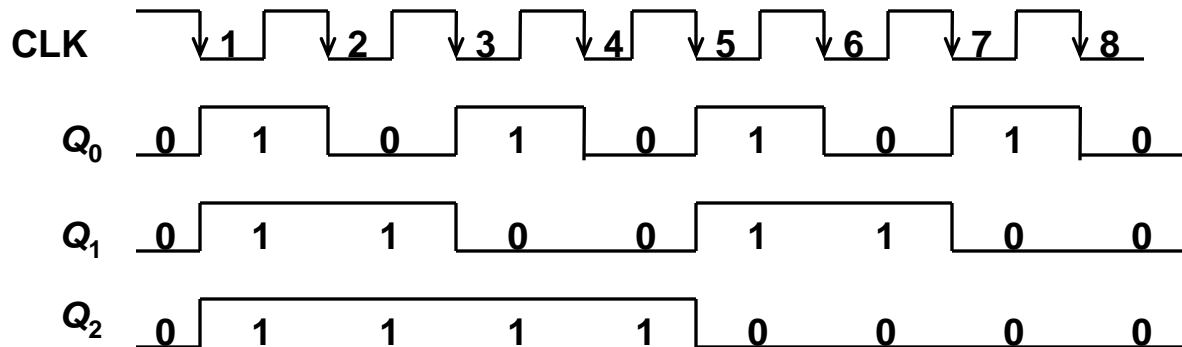
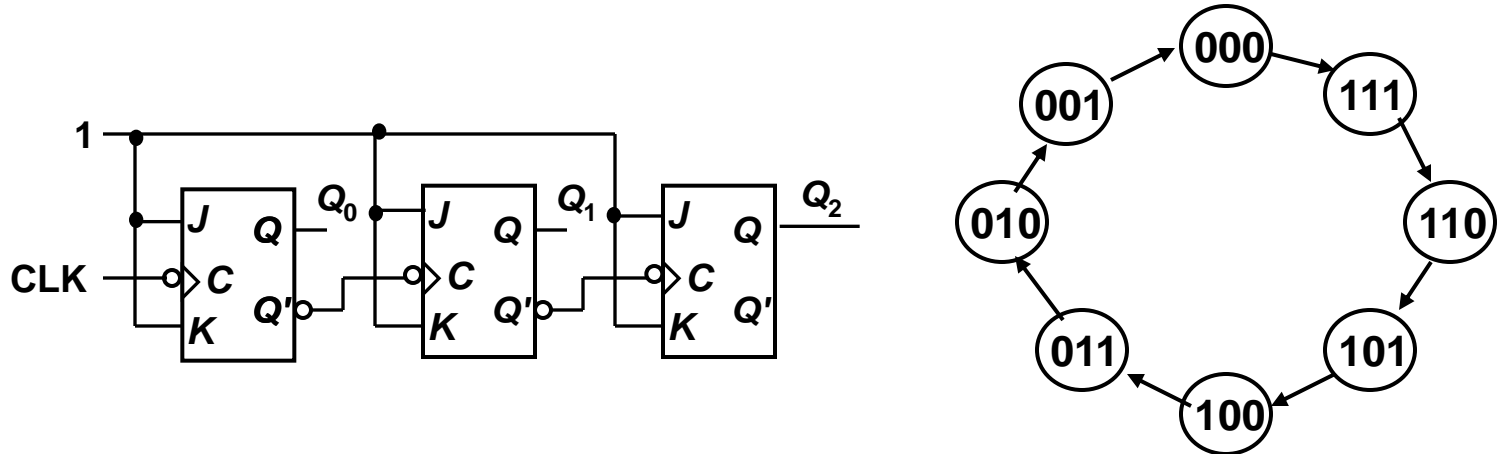
3-bit binary up counter



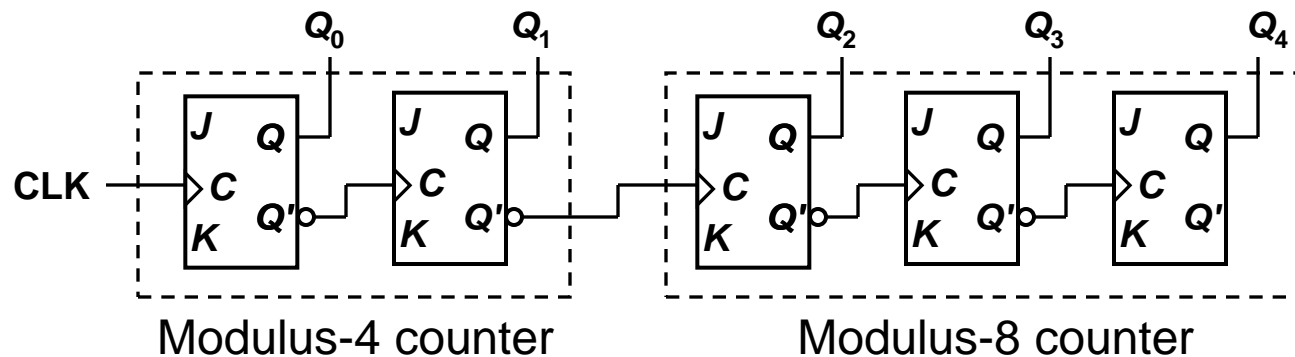
3-bit binary down counter



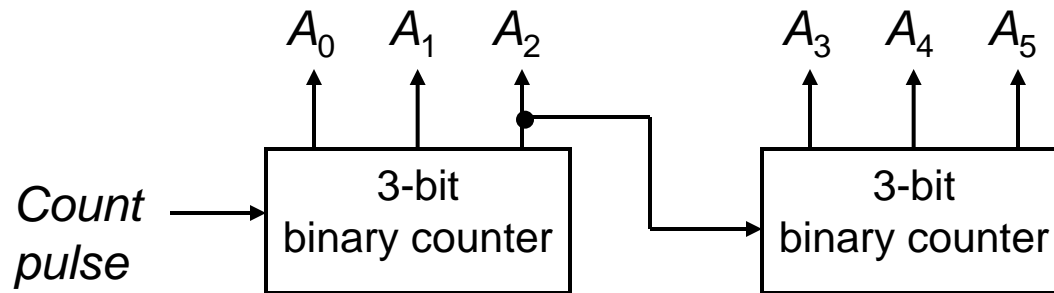
- Example: A 3-bit binary (MOD-8) down counter.



- Larger asynchronous (ripple) counter can be constructed by cascading smaller ripple counters.
- Connect last-stage output of one counter to the clock input of next counter so as to achieve higher modulus operation.
- Example: A modulus-32 ripple counter constructed from a modulus-4 counter and a modulus-8 counter.

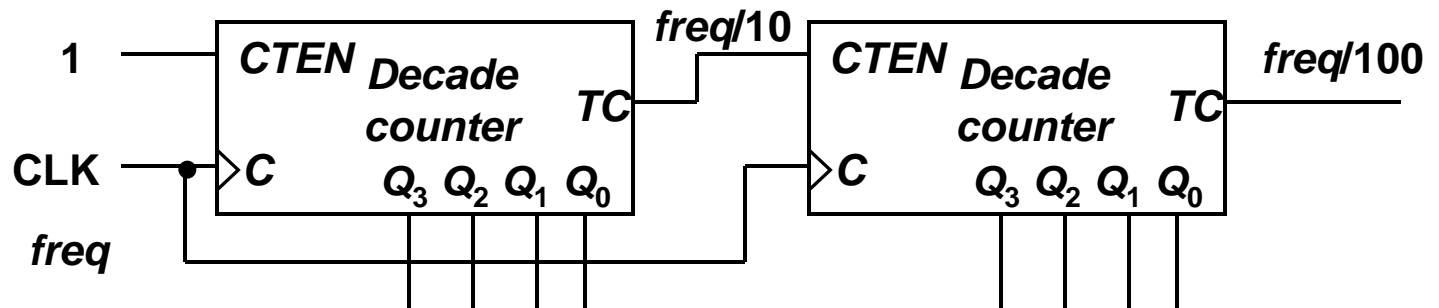


- Example: A 6-bit binary counter (counts from 0 to 63) constructed from two 3-bit counters.



$A_5$	$A_4$	$A_3$	$A_2$	$A_1$	$A_0$
0	0	0	0	0	0
0	0	0	0	0	1
0	0	0	:	:	:
0	0	0	1	1	1
0	0	1	0	0	0
0	0	1	0	0	1
:	:	:	:	:	:

- If counter is a not a binary counter, requires additional output.
- Example: A modulus-100 counter using two decade counters.



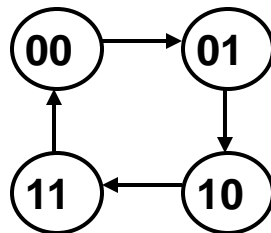
**$TC = 1$  when counter recycles to 0000**



# Synchronous (Parallel) Counters



- **Synchronous (parallel) counters:** the flip-flops are clocked at the same time by a common clock pulse.
- We can design these counters using the sequential logic design process (covered in Lecture #12).
- Example: 2-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J,K inputs).



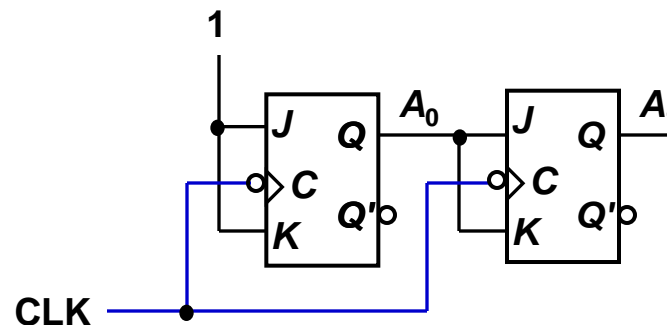
Present state		Next state		Flip-flop inputs	
$A_1$	$A_0$	$A_1^+$	$A_0^+$	$TA_1$	$TA_0$
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	1

- Example: 2-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J,K inputs).

Present state		Next state		Flip-flop inputs	
$A_1$	$A_0$	$A_1^+$	$A_0^+$	$TA_1$	$TA_0$
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	1

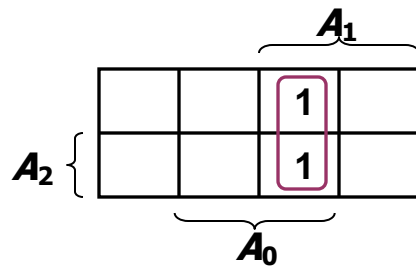
$$TA_1 = A_0$$

$$TA_0 = 1$$

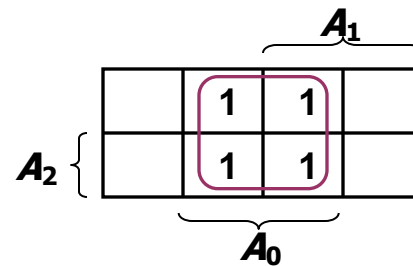


- Example: 3-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J, K inputs).

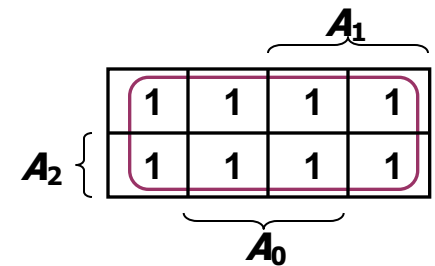
Present state			Next state			Flip-flop inputs		
$A_2$	$A_1$	$A_0$	$A_2^+$	$A_1^+$	$A_0^+$	$TA_2$	$TA_1$	$TA_0$
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1



$$TA_2 = A_1 \cdot A_0$$



$$TA_1 = A_0$$



$$TA_0 = 1$$







# Synchronous (Parallel) Counters



- Note that in a binary counter, the  $n^{\text{th}}$  bit (shown underlined) is always complemented whenever

$$\underline{0}11\dots11 \rightarrow \underline{1}00\dots00$$

$$\text{or } \underline{1}11\dots11 \rightarrow \underline{0}00\dots00$$

- Hence,  $X_n$  is complemented whenever

$$X_{n-1}X_{n-2} \dots X_1X_0 = 11\dots11.$$

- As a result, if T flip-flops are used, then

$$TX_n = X_{n-1} \cdot X_{n-2} \cdot \dots \cdot X_1 \cdot X_0$$

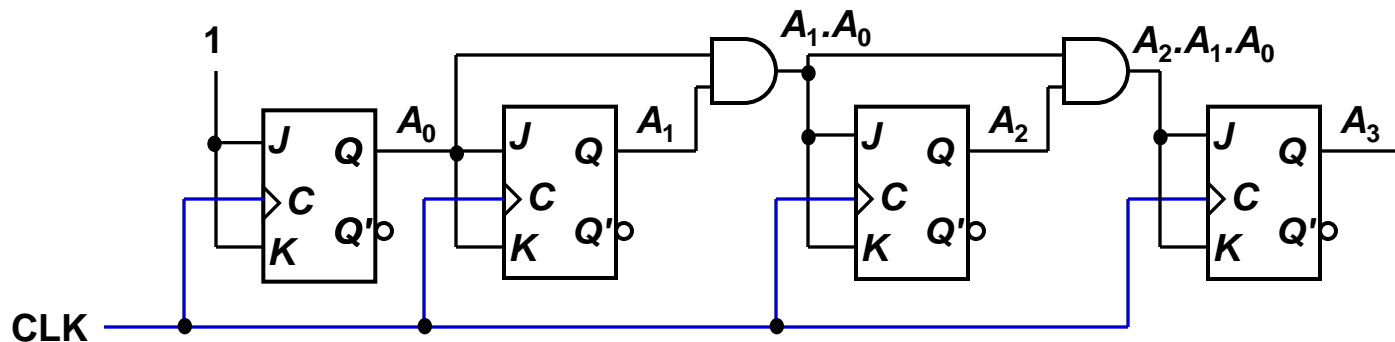
- Example: 4-bit synchronous binary counter.

$$TA_3 = A_2 \cdot A_1 \cdot A_0$$

$$TA_2 = A_1 \cdot A_0$$

$$TA_1 = A_0$$

$$TA_0 = 1$$





# Synchronous (Parallel) Counters



- Example: Synchronous decade/BCD counter.

Clock pulse	$Q_3$	$Q_2$	$Q_1$	$Q_0$
Initially	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10 (recycle)	0	0	0	0

$$T_0 = 1$$

$$T_1 = Q_3' \cdot Q_0$$

$$T_2 = Q_1 \cdot Q_0$$

$$T_3 = Q_2 \cdot Q_1 \cdot Q_0 + Q_3 \cdot Q_0$$





# Up/Down Synchronous Counters



- **Up/down synchronous counter:** a *bidirectional* counter that is capable of counting either up or down.
- An input (control) line  $Up/\overline{Down}$  (or simply  $Up$ ) specifies the direction of counting.
  - ❖  $Up/\overline{Down} = 1 \rightarrow$  Count upward
  - ❖  $Up/\overline{Down} = 0 \rightarrow$  Count downward



# Up/Down Synchronous Counters



- Example: A 3-bit up/down synchronous binary counter.

Clock pulse	<i>Up</i>	$Q_2$	$Q_1$	$Q_0$	<i>Down</i>
0		0	0	0	
1		0	0	1	
2		0	1	0	
3		0	1	1	
4		1	0	0	
5		1	0	1	
6		1	1	0	
7		1	1	1	

$$TQ_0 = 1$$

$$TQ_1 = (Q_0 \cdot Up) + (Q_0' \cdot Up')$$

$$TQ_2 = (Q_0 \cdot Q_1 \cdot Up) + (Q_0' \cdot Q_1' \cdot Up')$$

Up counter

$$TQ_0 = 1$$

$$TQ_1 = Q_0$$

$$TQ_2 = Q_0 \cdot Q_1$$

Down counter

$$TQ_0 = 1$$

$$TQ_1 = Q_0'$$

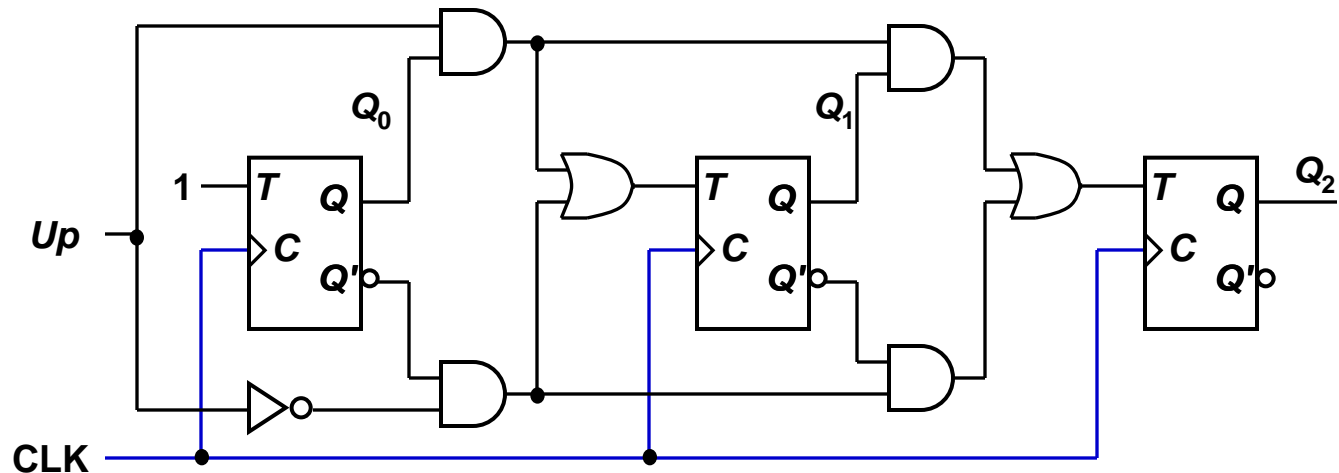
$$TQ_2 = Q_0' \cdot Q_1'$$

- Example: A 3-bit up/down synchronous binary counter (cont'd).

$$TQ_0 = 1$$

$$TQ_1 = (Q_0 \cdot Up) + (Q_0' \cdot Up')$$

$$TQ_2 = (Q_0 \cdot Q_1 \cdot Up) + (Q_0' \cdot Q_1' \cdot Up')$$

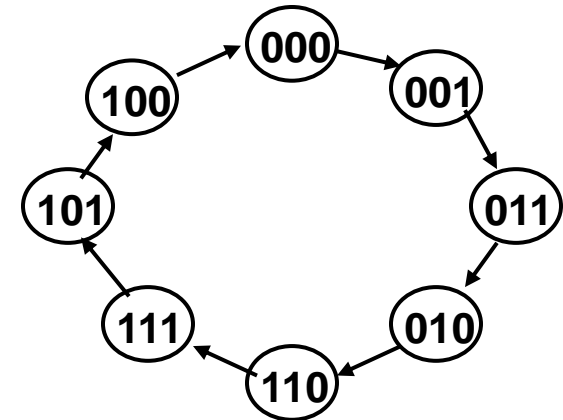




# Designing Synchronous Counters



- Covered in Lecture #12.
- Example: A 3-bit Gray code counter (using JK flip-flops).



Present state			Next state			Flip-flop inputs					
$Q_2$	$Q_1$	$Q_0$	$Q_2^+$	$Q_1^+$	$Q_0^+$	$JQ_2$	$KQ_2$	$JQ_1$	$KQ_1$	$JQ_0$	$KQ_0$
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	1	0	X	1	X	X	0
0	1	0	1	1	0	1	X	X	0	0	X
0	1	1	0	1	0	0	X	X	0	X	1
1	0	0	0	0	0	X	1	0	X	0	X
1	0	1	1	0	0	X	0	0	X	X	1
1	1	0	1	1	1	X	0	X	0	1	X
1	1	1	1	0	1	X	0	X	1	X	0



- 3-bit Gray code counter: flip-flop inputs.

		$Q_1 Q_0$			
$Q_2$		00	01	11	10
0					1
1		X	X	X	X

$JQ_2 = Q_1 \cdot Q_0'$

		$Q_1 Q_0$			
$Q_2$		00	01	11	10
0			1	X	X
1				X	X

$JQ_1 = Q_2' \cdot Q_0$

		$Q_1 Q_0$			
$Q_2$		00	01	11	10
0		1	X	X	
1			X	X	1

$JQ_0 = Q_2 \cdot Q_1 + Q_2' \cdot Q_1'$   
 $= (Q_2 \oplus Q_1)'$

		$Q_1 Q_0$			
$Q_2$		00	01	11	10
0		X	X	X	X
1		1			

$KQ_2 = Q_1' \cdot Q_0'$

		$Q_1 Q_0$			
$Q_2$		00	01	11	10
0		X	X		
1		X	X	1	

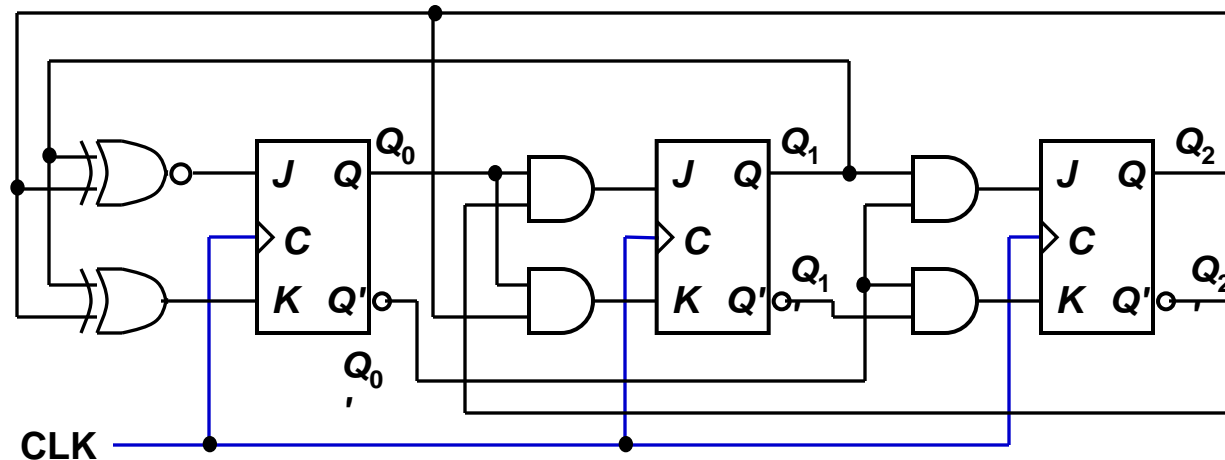
$KQ_1 = Q_2 \cdot Q_0$

		$Q_1 Q_0$			
$Q_2$		00	01	11	10
0		X		1	X
1		X	1		X

$KQ_0 = Q_2 \cdot Q_1' + Q_2' \cdot Q_1$   
 $= Q_2 \oplus Q_1$

- 3-bit Gray code counter: logic diagram.

$$\begin{array}{lll}
 JQ_2 = Q_1 \cdot Q_0' & JQ_1 = Q_2' \cdot Q_0 & JQ_0 = (Q_2 \oplus Q_1)' \\
 KQ_2 = Q_1' \cdot Q_0' & KQ_1 = Q_2 \cdot Q_0 & KQ_0 = Q_2 \oplus Q_1
 \end{array}$$



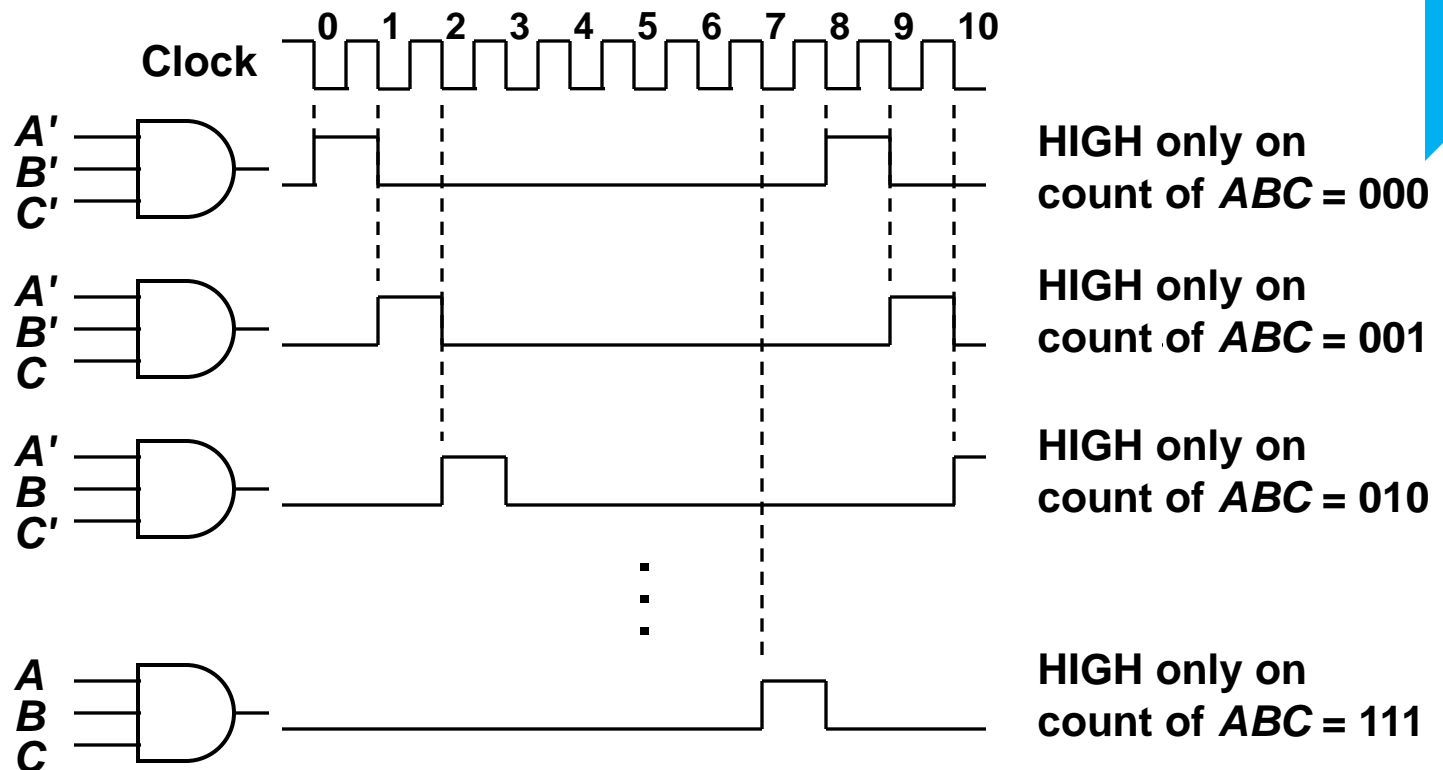


# Decoding A Counter



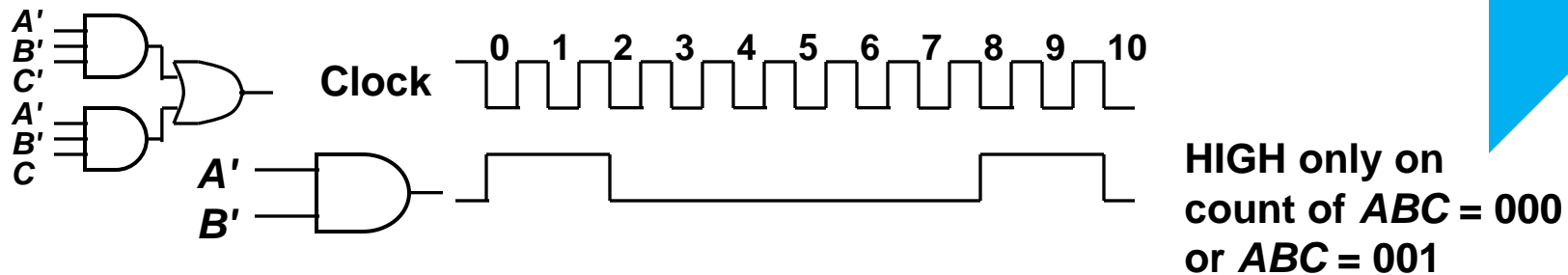
- **Decoding a counter** involves determining which state in the sequence the counter is in.
- Differentiate between *active-HIGH* and *active-LOW* decoding.
- Active-HIGH decoding: output HIGH if the counter is in the state concerned.
- Active-LOW decoding: output LOW if the counter is in the state concerned.

- Example: MOD-8 ripple counter (active-HIGH decoding).

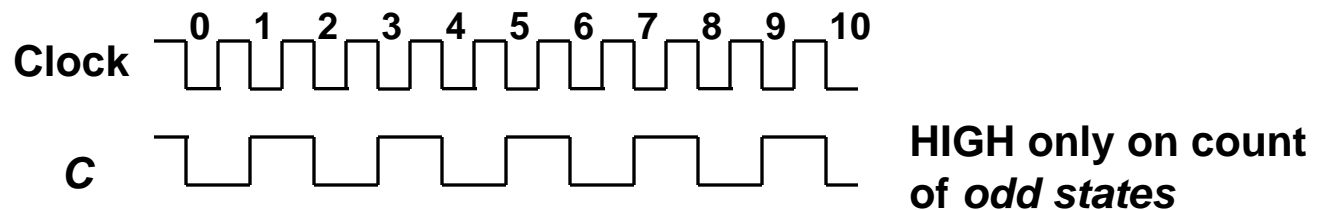


# Decoding A Counter

- Example: To detect that a MOD-8 counter is in state 0 (000) or state 1 (001).



- Example: To detect that a MOD-8 counter is in the odd states (states 1, 3, 5 or 7), simply use C.



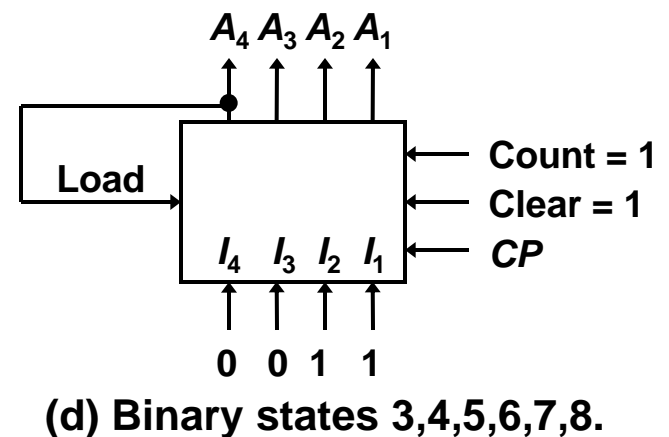
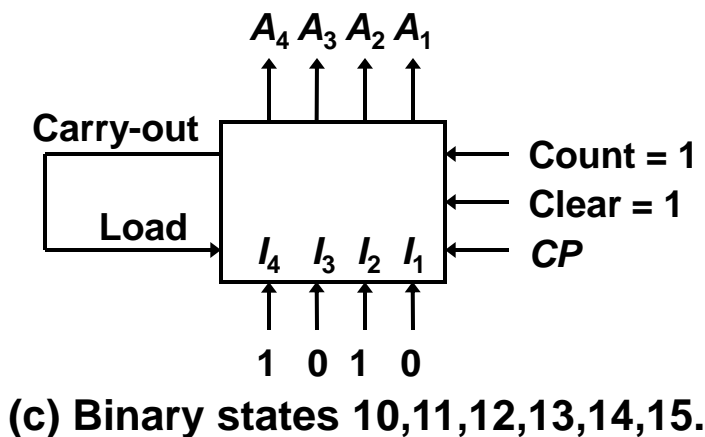
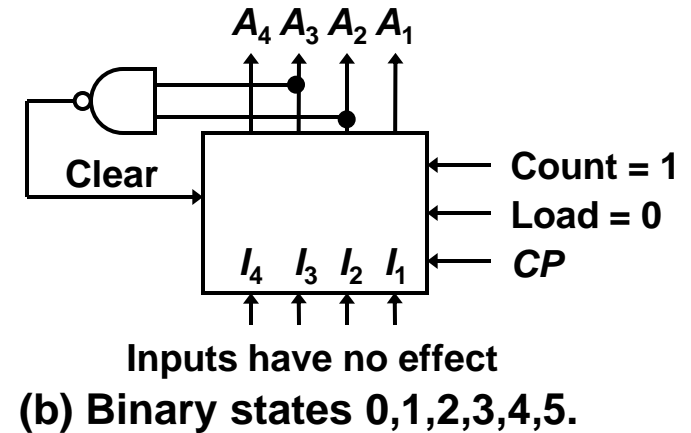
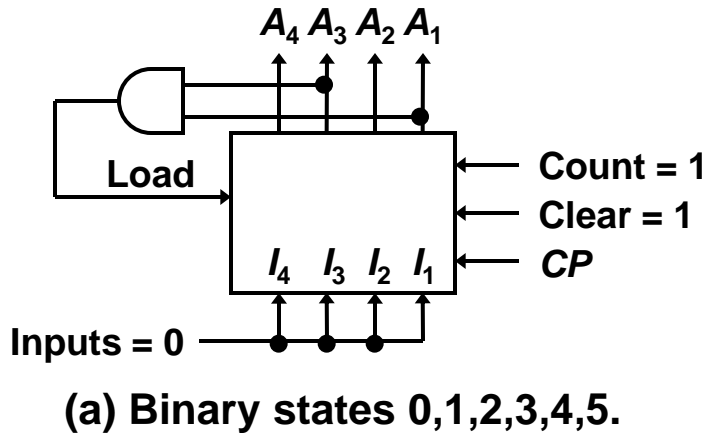


# Counters with Parallel Load



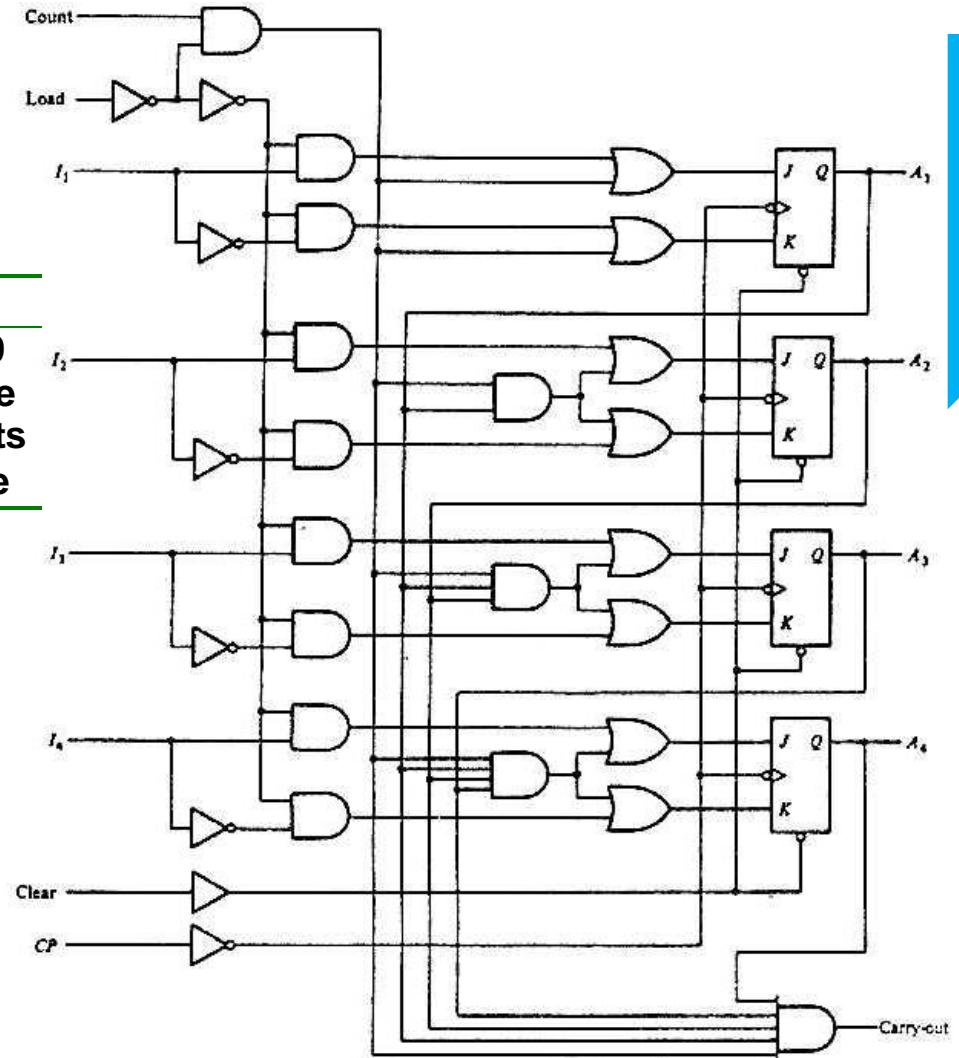
- Counters could be augmented with parallel load capability for the following purposes:
  - ❖ To start at a different state
  - ❖ To count a different sequence
  - ❖ As more sophisticated register with increment/decrement functionality.

- Different ways of getting a MOD-6 counter:



- 4-bit counter with parallel load.

Clear	CP	Load	Count	Function
0	X	X	X	Clear to 0
1	X	0	0	No change
1	↑	1	X	Load inputs
1	↑	0	1	Next state





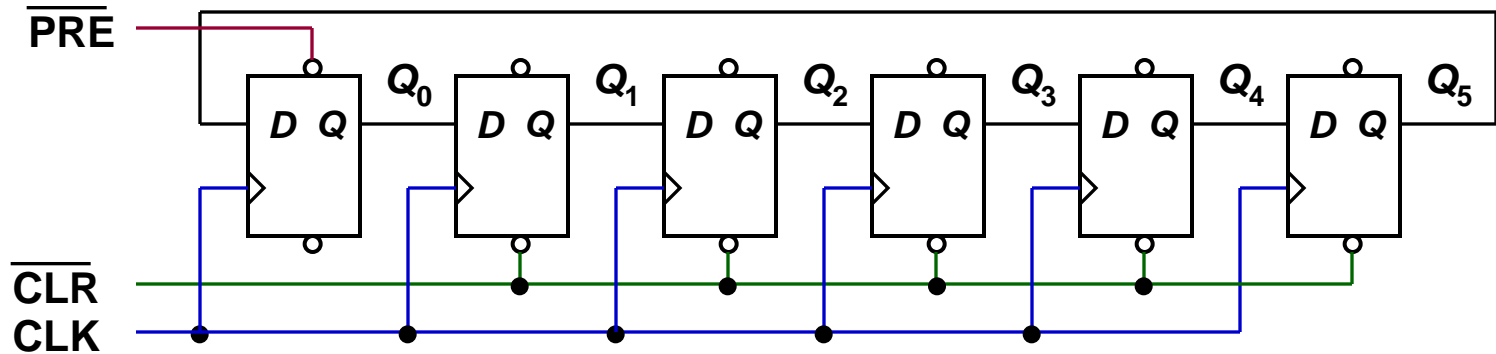


# Ring Counters

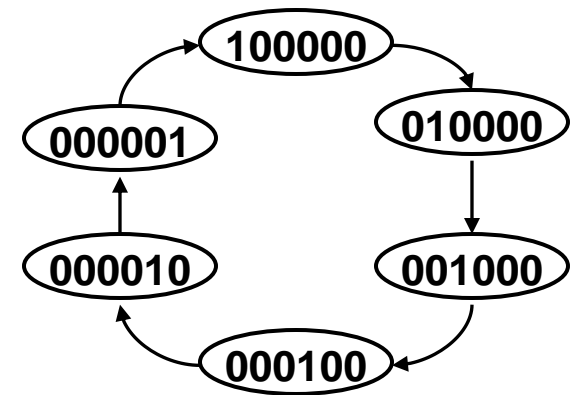


- One flip-flop (stage) for each state in the sequence.
- The output of the last stage is connected to the D input of the first stage.
- An  $n$ -bit ring counter cycles through  $n$  states.
- No decoding gates are required, as there is an output that corresponds to every state the counter is in.

- Example: A 6-bit (MOD-6) ring counter.



Clock	Q <sub>0</sub>	Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>	Q <sub>5</sub>
0	1	0	0	0	0	0
1	0	1	0	0	0	0
2	0	0	1	0	0	0
3	0	0	0	1	0	0
4	0	0	0	0	1	0
5	0	0	0	0	0	1



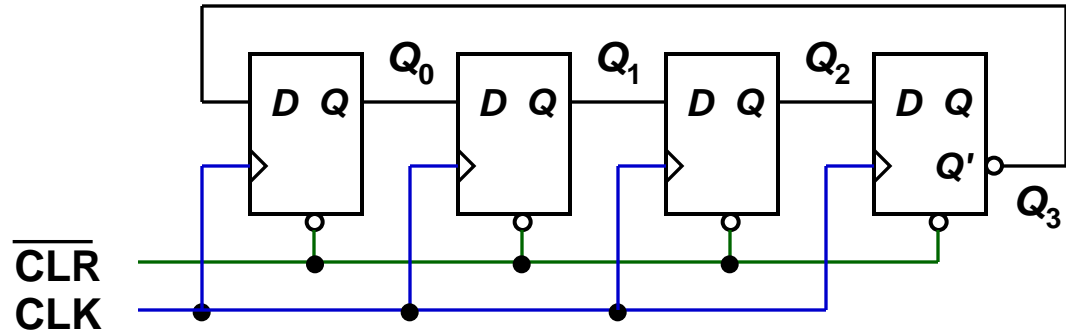


# Johnson Counters

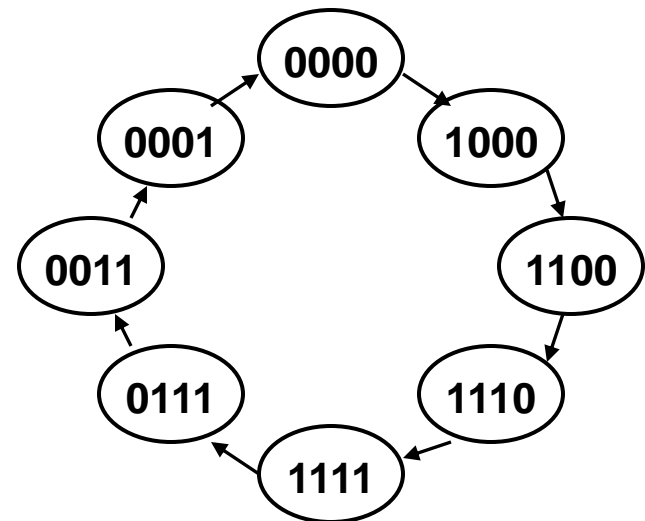


- The complement of the output of the last stage is connected back to the D input of the first stage.
- Also called the *twisted-ring counter*.
- Require fewer flip-flops than ring counters but more flip-flops than binary counters.
- An  $n$ -bit Johnson counter cycles through  $2n$  states.
- Require more decoding circuitry than ring counter but less than binary counters.

- Example: A 4-bit (MOD-8) Johnson counter.

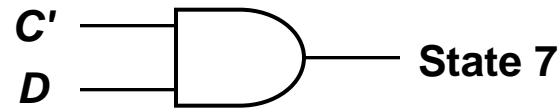
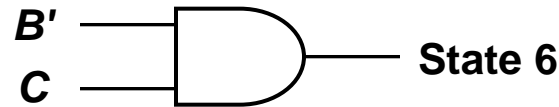
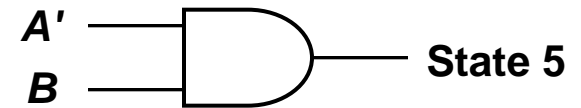
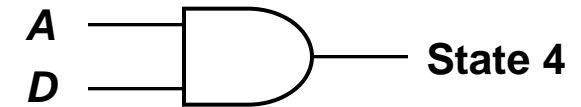
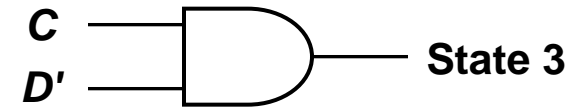
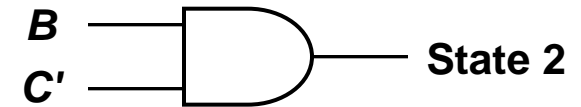
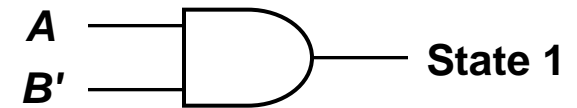
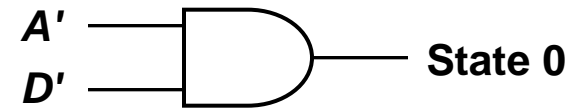


Clock	$Q_0$	$Q_1$	$Q_2$	$Q_3$
0	0	0	0	0
1	1	0	0	0
2	1	1	0	0
3	1	1	1	0
4	1	1	1	1
5	0	1	1	1
6	0	0	1	1
7	0	0	0	1



- Decoding logic for a 4-bit Johnson counter.

Clock	A	B	C	D	Decoding
0	0	0	0	0	$A'.D'$
1	1	0	0	0	$A.B'$
2	1	1	0	0	$B.C'$
3	1	1	1	0	$C.D'$
4	1	1	1	1	$A.D$
5	0	1	1	1	$A'.B$
6	0	0	1	1	$B'.C$
7	0	0	0	1	$C'.D$





Thank  
you

