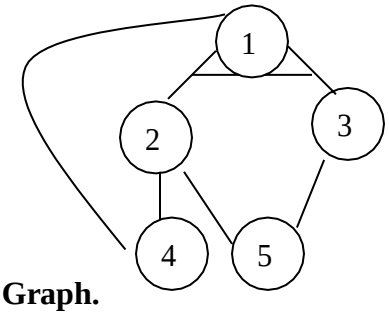
UNIT IV: GRAPHS

1. Define Graph with illustration.

A Graph is defined as G = (V, E) where V is the set of vertices (nodes) and E is the set of edges (arcs) connecting the vertices. An edge is represented as a pair of vertices (u,v).

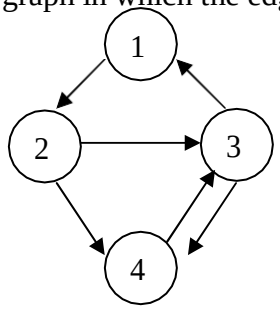


V = {1, 2, 3, 4, 5}

E = {(1, 2),(1, 3),(1,4),(2,3)
(2,4),(2,5),(3,5)}

2. Define Directed Graph.

A Directed graph is a graph in which the edges are directed. It is also called Digraph.



V={1,2,3,4}

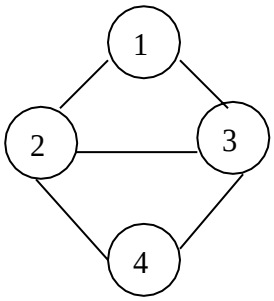
E={(1,2),(2,3),(2,4)
(3,1),(3,4),(4,3)}

3. Mention the ways of representing a graph?

- Adjacency Matrix representation
- Adjacency List representation

4. What do you mean by Undirected Graph?

Edges in the graph do not have directions marked. Such graphs are referred to as undirected graphs.



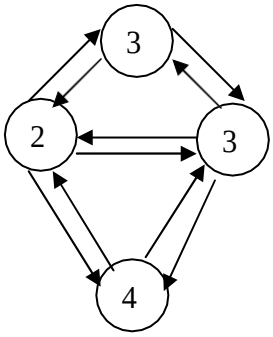
V={1,2,3,4}

E={(1,2),(2,1),(2,3),(3,2),(2,4),

(4,2),(3,4),(4,3),(3,1),(1,3)}

5. Define Symmetric Digraph.

Every edge has an edge in the reverse direction i.e for every edge (U,V) there is an edge (V,U). Such type of graph is called Symmetric Digraph.



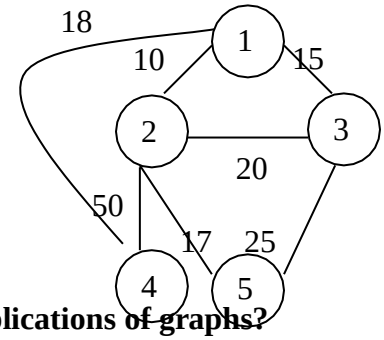
V={1,2,3,4}

E={(1,2),(2,1),(2,3),(3,2)

(2,4),(4,2),(3,4),(4,3),(3,1),(1,3)}

6. What do you mean by weighted graph?

Weighted graphs are such graphs where the edges are associated with weights. These weights are used to mark the importance of edges in representing a problem. Ex. Road map represented as graph where the weight is the distance between two places.



V={1,2,3,4,5}

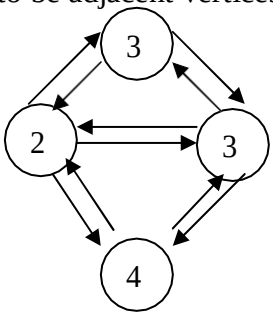
E={(1,2),(1,3),(1,4),(2,3)
(2,4),(2,5),(3,5)}

7. What are the applications of graphs?

- Transport systems
- Computer Networks
- Electric circuits

8. Define adjacent vertices.

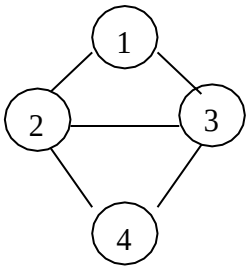
Two vertices are said to be adjacent vertices if there is an edge between them.



Vertices 1&2, 2&3, 2&4, 3&4 are adjacent vertices.

9. Define path.

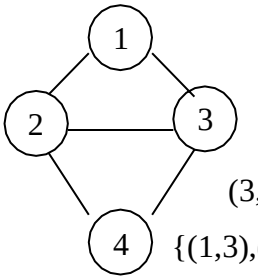
A Path between vertices (u,v) is a sequence of edges which connects vertices u & v.



The path between the vertices 1 and 4 is 1-2-4 or 1-3-4

10. Define Length of the Path.

Length of the path is the number of edges in a path which is equal to N-1. N represents the number of vertices.



The Length of the path 1 -4 {(1, 2), (2, 3), (3,4) } is 3

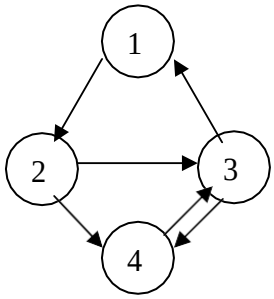
{(1,3),(3,4)} is 2 and also {(1,2),(2,4)} is 2.

11. What is a forest?

A forest may be defined as an acyclic graph in which every node has one or no of predecessors. A tree may be defined as a forest in which only a single node called root has no predecessors.

12. Define cycle.

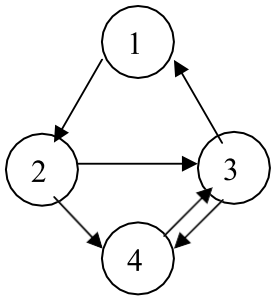
A Cycle in a graph is a path that starts and ends at the same vertex. i.e. path in which the first and last vertices are same.



Ex: {(3, 1), (1, 2), (2, 4), (4, 3)} is a Cycle.

13. Define simple cycle.

Simple cycle means the vertex should not be repeated and the first and last vertex should be the same.

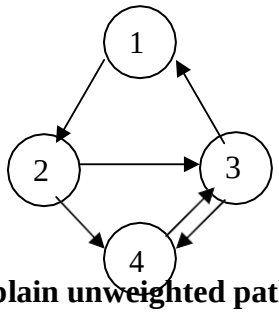


Example: 1-2-4-3-1 is a simple cycle

1-2-3-4-3-1 is not a simple Cycle

14. Define Simple Path.

All vertices are distinct, except that the first and last may or may not be the same.



Example: 1-2-4-3-1 is a simple path

1-2-3-4-3-1 is not a simple path

1-2-4-3 is a simple path

15. Explain unweighted path length

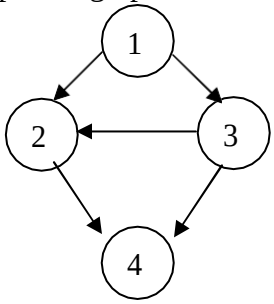
Unweighted path length is the number of edges on the path namely, N-1(where N is the number of vertices).

16. What is a tree edge?

Traversal from one vertex to the next vertex in a graph is called as a tree edge.

17. Define acyclic graph.

An acyclic graph is a graph with no cycle.

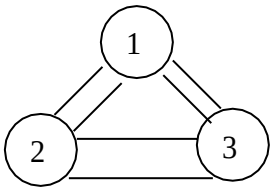


18. What do you mean by a strongly connected graph?

A Directed graph is strongly connected if there is a path from every vertex to every other vertex.

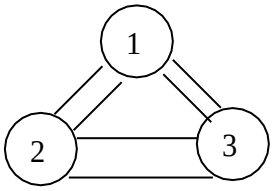
19. What do you mean by weakly connected graph?

If the directed graph is not strongly connected, but the underlying graph (without direction to the arcs) is connected, then the graph is said to be weakly connected.



20. Define complete graph.

A graph in which there is an edge between every pair of vertices.



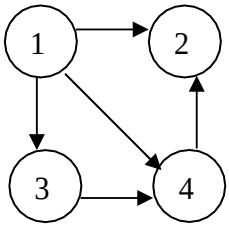
21. Explain weighted path length.

The cost of the path $V_1, V_2 \ldots V_N$ is $\sum_{i=1}^{N-1} C_{i,\, i+1}$. This is referred to as the weighted path length.

22. What do you mean by Adjacency Matrix representation?

The adjacency matrix M of a graph G = (V, E) is a matrix of order $V_i \times V_j$ and the elements of the unweighted graph M are defined as, M [i] [j] = 1, if $(V_i, V_j) \in E$ = 0 Otherwise, For a weighted graph the elements of M are defined as M [i] [j] = W_{ij}, if $(V_i, V_j) \in E$ and W_{ij} is the weight of edge (V_i, V_j) = 0 Otherwise

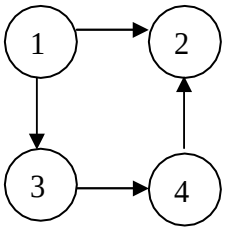
Example:

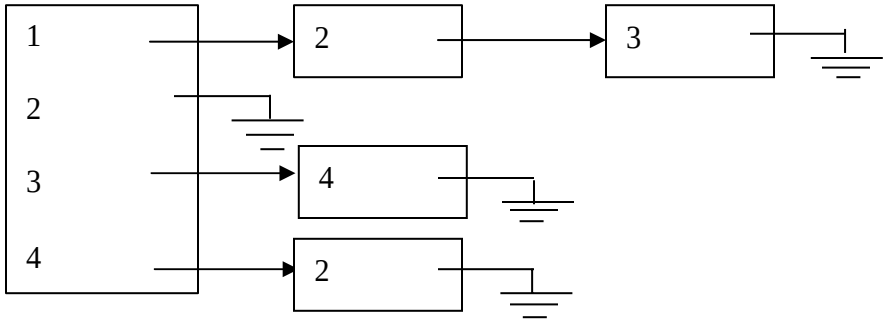


	1	2	3	4
1	0	1	1	1
2	0	0	0	1
3	0	0	0	1
4	0	1	0	0

23. What do you mean by adjacency List representation?

It is an array of linked list, for each vertex a linked list of all adjacent vertices is maintained. It consists of a table with the no. of entries for each vertex for each entry a linked List is initiated for the vertices adjacent to the corresponding table entry.





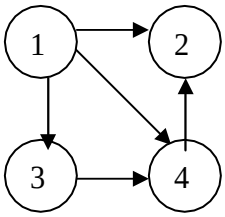
24. What are the conditions for a graph to become a tree?

A graph is a tree if it has two properties. i. If it is a connected graph. ii. There should not be any cycles in the graph.

25. What do you mean by Indegree and Outdegree of a graph?

- Indegree of a vertex in a graph is the number of incoming edges.
- Outdegree of a vertex is the number of edges that leaves the vertex.

Example:



Indegree for the vertex 1 is 0, vertex 2 is 2, vertex 3 is 1, and vertex 4 is 2

Outdegree for the vertex 1 is 3, vertex 2 is 0, vertex 3 is 1, and vertex 4 is 1.

26. Define Topological sort.

Topological sort is defined as an ordering of vertices in a directed acyclic graph. Such that if there is a path from V_i to V_j, then V_j appears after V_i in the ordering.

27. Explain the principle of topological sort.

- Find the vertex with no incoming edge.
- Print the vertex and remove it along with its edges from the graph.
- Apply the same strategy to the rest of the graph.
- Finally all recorded vertices give topological sorted list.

28. What is the disadvantage of topological sort?

Ordering of vertices is not possible if the graph is a cyclic graph. because if there are two vertices v and w on the cycle, v proceeds w and w proceeds v, so ordering not unique.

29. What is the running time for topological sort?

The running time of the algorithm for topological sort is $O(V^2)$.For the algorithm using Queue, the running time is O (|E| +|V|) if adjacency list are used.

30. What is a back edge?

The possibility of reaching an already marked vertex is indicated by a dashed line, in a graph is called as back edge.

31. State the shortest path problem or single source shortest path problem.

Given as input a weighted graph or an unweighted graph G= (V, E) and a distinguished vertex s , the shortest path problem is to find the shortest weighted or unweighted path from s to every other vertex.

32. What do you mean by Negative edge?

Negative edge means a graph having at least one edge with a negative weight.

33. Give examples for problems solved by shortest path algorithm.

- Cheapest way of sending electronic news from one computer to another.
- To compute the best route

34. What is the running time for the weighted and unweighted shortest path?

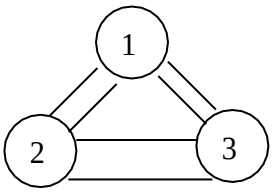
- The running time for the weighted shortest path is O (|E| + | V|)
- The running time for the Unweighted shortest path is O (|E| log |V|)

35. What is the advantage of unweighted shortest path algorithm?

There is no calculation with weights. Only the number of edges on the shortest path is found.

36. What is a connected graph?

An undirected graph is connected if there is a path from every vertex to every other vertex. A directed graph with this property is called as strongly connected graph. If a directed graph is not strongly connected but the underline graph without direction is connected it is called as a weakly connected graph.



37. What are the two traversal strategies used in traversing a graph?

a. Breadth first search/traversals (BFS/BFT)

b. Depth first search /traversals (DFS/DFT)

38. What is a minimum spanning tree?

A minimum spanning tree of an undirected graph G is a tree formed from graph edges that connects all the vertices of G at the lowest total cost.

39. What is an undirected acyclic graph?

When every edge in an acyclic graph is undirected, it is called an undirected acyclic graph. It is also called as undirected forest.

40. What is a minimum spanning tree?

A minimum spanning tree of an undirected graph G is a tree formed from graph edges. That connects all the vertices of G at lowest total cost.

41. What is a single source shortest path problem?

Given as an input, a weighted graph, G=<V,E> and a distinguished vertex ‘S’ as the source vertex. Single source shortest path problem finds the shortest weighted path from s to every other vertex in G.

42. Explain about Unweighted shortest path

Single source shortest path finds the shortest path from the source to each and every vertex present in a unweighted graph. Here no cost is associated with the edges connecting the vertices. Always unit cost is associated with each edge.

43. Explain about Weighted shortest path

Single source shortest path finds the shortest path from the source to each and Every vertex present in a weighted graph. In a weighted graph some cost is always associated with the edges connecting the vertices.

44. What are the methods to solve minimum spanning tree?

a) Prim's algorithm b) Kruskal's algorithm

45. Explain briefly about Prim's algorithm

Prim's algorithm creates the spanning tree in various stages. At each stage, a node is picked as the root and an edge is added and thus the associated vertex along with it.

46. Define a depth first spanning tree.

The tree that is formulated by depth first search on a graph is called as depth first spanning tree. The depth first spanning tree consists of tree edges and back edges.

47. What is activity node in graph?

The activity-node graph is a vertex-weighted graph. An activity involves both synchronization and branching constructs, similar to but more powerful than a traditional flow chart, which only supports sequential and branching constructs.

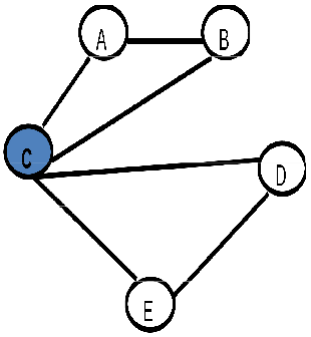
48. Define Euler tours/circuits?

An Euler cycle, Euler circuit or Euler tour in an undirected graph is a cycle that uses each edge exactly once. If such a cycle exists, the graph is called unicursal. While such graphs are Euler graphs, not every Euler graph possesses an Euler cycle.

49. Define articulation points.

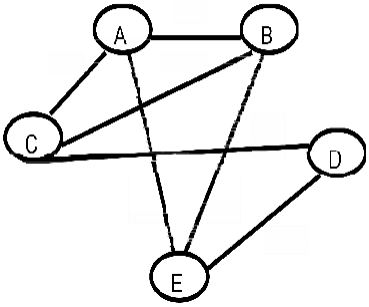
If a graph is not biconnected, the vertices whose removal would disconnect the graph are known as articulation points. We can compute articulation points using depth-first search and a special numbering of the vertices in the order of their visiting. For example

C is an articulation point

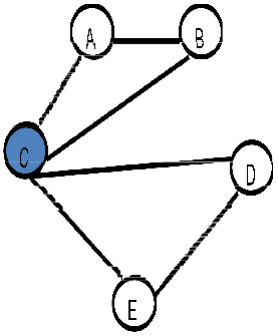


50. What is biconnected graph?

A graph is biconnected, if there are no vertices whose removal will disconnect the graph. For example.



Biconnected



Not biconnected

51. Distinguish BFS and DFS.

BFS	DFS
BFS removes them from the beginning, which results in maintaining the list as a queue	DFS removes them from the end, maintaining the list as a stack.
For BFS in directed graphs each edge of the graph either: • connects two vertices at the same level • Goes down exactly one level. • Goes up any number of levels.	For DFS, each edge either connects an ancestor to : • a descendant • a descendant to an ancestor • One node to a node in a previously visited sub tree.

52. List the Applications of Graphs.

- Computer Networks
- Transport

- Electronic Circuit