SNS COLLEGE OF ENGINEERING
(Autonomous)
DEPARTMENT OF ELECTRONICS AND COMMUNICATIOENGINEERING

Quine Mc Clusky(Tabulation) method

Quine Mc Clusky method

A systematic simplification procedure to reduce a minterm expansion to a minimum sum of products.

Use XY + XY’ = X to eliminate as many as literals as possible.

The resulting terms = prime implicants.

Use a prime implicant chart to select a minimum set of prime implicants.

Determination of Prime Implicants

√Eliminate literals

Two terms can be combined if they differ in exactly one variable.

AB'CD' + AB'CD = AB'C

1 0 1 0 + 1 0 1 1 = 1 0 1

X Y X Y' X

A'BC'D + A'BCD' (won't combine)

0 1 0 1 + 0 1 1 0 (check # of 1's)

We need to compare and combine whenever possible.

Sorting to Reduce Comparisons

√Sort into groups according to the number of 1's.

F(a,b,c.d) = Σm(0,1,2,5,6,7,8,9,10,14)

Group	0	0	0000
Group	1	1	0001
		2	0010
		8	1000
Group	2	5	0101
		6	0110
		9	1001
		10	1010
Group	3	7	0111
		14	1110

No need for comparisons

(1) Terms in nonadjacent group

(2) Terms in the same group

Comparison of adjacent groups

Use X + X = X repeatedly between adjacent groups

Those combined are checked off.

Combine terms that have the same dashes and differ one in the number of 1's. (for column II and column III)

$$f = \underset{(1,\,5)}{a'c'd} + \underset{(5,\,7)}{a'bd} + \underset{(6,\,7)}{a'bc} + \underset{(0,\,1,\,8,\,9)}{b'c'} + \underset{(0,\,2,\,8,\,10)}{b'd'} + \underset{(2,\,6,\,10,\,14)}{cd'}$$

$$f = a'bd + b'c' + cd'$$

Determination of Prime Implicants

Column I

group 0	0	0000	✓
group 1	1	0001	✓
	2	0010	✓
	8	1000	✓
group 2	5	0101	✓
	6	0110	✓
	9	1001	✓
	10	1010	✓
group 3	7	0111	✓
	14	1110	✓

Column II

0, 1	000–	✓
0, 2	00–0	✓
0, 8	–000	✓
1, 5	0–01	
1, 9	–001	✓
2, 6	0–10	✓
2, 10	–010	✓
8, 9	100–	✓
8, 10	10–0	✓
5, 7	01–1	
6, 7	011–	
6, 14	–110	✓
10, 14	1–10	✓

Column III

0, 1, 8, 9	–00–
0, 2, 8, 10	–0–0
~~0, 8, 1, 9~~	~~–00–~~
~~0, 8, 2, 10~~	~~–0–0~~
2, 6, 10, 14	––10
~~2, 10, 6, 14~~	~~––10~~

Prime Implicants

The terms that have not been checked off are called prime implicants.

f = 0-01 + 01-1+011- + -00- + -0-0 + --10

= <u>a'c'd</u> + a'bd + <u>a'bc</u> + b'c' + <u>b'd'</u> + cd'

Each term has a minimum number of literals, but minimum SOP for f:

f = a'bd + b'c' + cd'

(a'bd, cd' => a'bc)(a'bd, b'c' => a'c'd) (b'c', cd' => b'd')

Definition of Implicant

Definition

Given a function of F of n variables, a product term P is an implicant of F iff for every combination of values of the n variables for which P = 1, F is also equal to 1.

Every minterm of F is an implicant of F.

Any term formed by combining two or more minterms is an implicant.

If F is written in SOP form, every product term is an implicant.

Example: f(a,b,c) = a'b'c' + ab'c' + ab'c + abc = b'c' + ac

If a'b'c' = 1, then F = 1, if ac = 1, then F = 1. a'b'c' and ac are implicants.

If bc = 1, (but a = 0), F = 0, so bc is not an implicant of F.

Definition of Prime Implicant

Definition

A prime implicant of a function F is a product term implicant which is no longer an implicant if any literal is deleted from it.

Example: f(a,b,c) = a’b’c’ + ab’c’ + ab’c + abc = b’c’ + ac

Implicant a’b’c’ is not a prime implicant. Why? If a’ is deleted, b’c’ is still an implicant of F.

b’c’ and ac are prime implicants.

Each prime implicant of a function has a minimum number of literals that no more literals can be eliminated from it or by combining it with other terms.

Quine McClusky Procedure

QM procedure:

Find all product term implicants of a function

Combine non-prime implicants.

Remaining terms are prime implicants.

A minimum SOP expression consists of a sum of some (not necessarily all) of the prime implicants of that function.

We need to select a minimum set of prime implicants.

If an SOP expression contains a term which is not a prime implicant, the SOP cannot be minimum.

Prime Implicant Chart

Chart layout

Top row lists minterms of the function

All prime implicants are listed on the left side.

Place x into the chart according to the minterms that form the corresponding prime implicant.

Essential prime implicant

If a minterm is covered only by one prime implicant, that prime implicant is called essential prime implicant. (9 & 14).

Essential prime implicant must be included in the minimum sum of the function.

		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	$b'c'$	X	X					X	(X)		
(0, 2, 8, 10)	$b'd'$	X		X				X		X	
(2, 6, 10, 14)	cd'			X		X				X	(X)
(1, 5)	$a'c'd$		X		X						
(5, 7)	$a'bd$				X		X				
(6, 7)	$a'bc$					X	X				

Selection of Prime Implicants

√Cross out the row of the selected essential prime implicants

√The columns which correspond to the minterms covered by the selected prime implicants are also crossed out.

√Select a prime implicant that covers the remaining columns. This prime implicant is not essential.

		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	$b'c'$	X	X					X	X		
(0, 2, 8, 10)	$b'd'$	X		X				X		X	
(2, 6, 10, 14)	cd'			X		X				X	X
(1, 5)	$a'c'd$		X		X						
(5, 7)	$a'bd$				X		X				
(6, 7)	$a'bc$					X	X				

A Cyclic Prime Implicant Chart

0 000
1 001
2 010
5 101
6 110
7 111

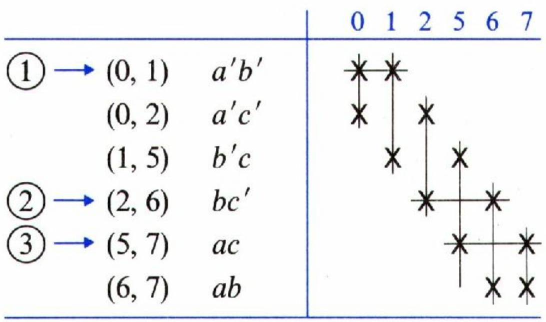
All checked off

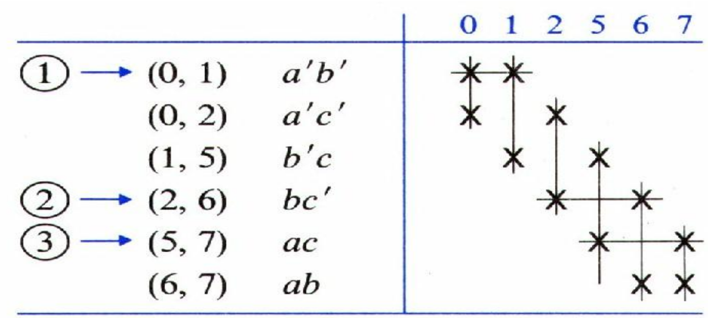
0,1 00-
0,2 0-0
1,5 -01
2,6 -10
5,7 1-1
6,7 11-

Two or more X's in every column.

$F = \Sigma m(0,1,2,5,6,7)$

F = a'b' + bc' + ac. (by try and error). No guarantee for this to be minimum.



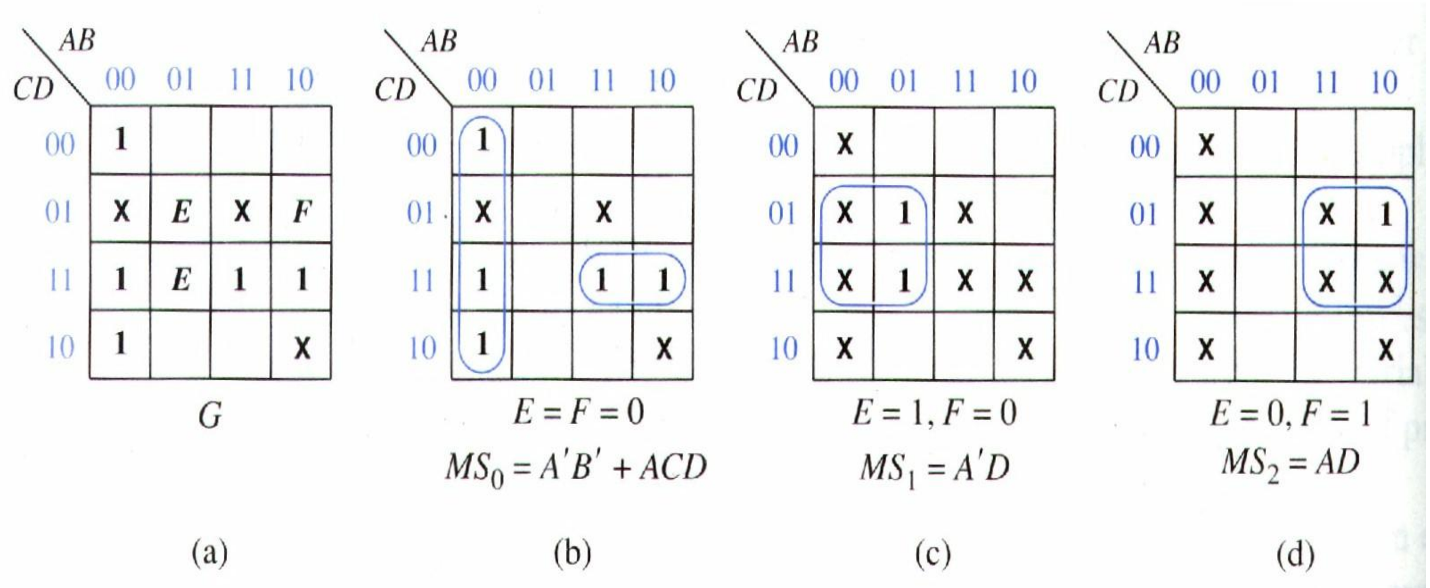


Simplification Using Map-Entered Variables

Extend K-map for more variables.

When E appears in a square, if E = 1, then the corresponding minterm is present in the function G.

$G(A,B,C,D,E,F) = m_0 + m_2 + m_3 + Em_5 + Em_7 + Fm_9 + m_{11} + m_{15}$ + (don't care terms)



Map-Entered Variable

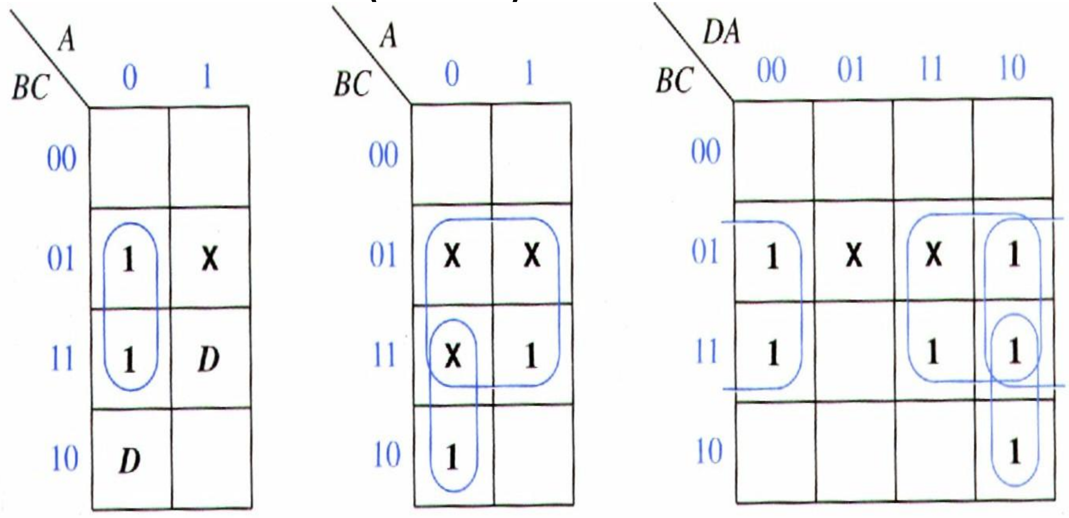
F(A,B,C,D) = A'B'C + A'BC + A'BC'D + ABCD + (AB'C), (don't care)

Choose D as a map-entered variable.

When D = 0, F = A'C (Fig. a) When D = 1, F = C + A'B (Fig. b)

two 1's are changed to x's since they are covered in Fig. a.

F = A'C + D(C+A'B) = A'C + CD + A'BD



(a) (b) (c)

General View for Map-Entered Variable Method

Given a map with variables P_1, P_2 etc, entered into some of the squares, the minimum SOP form of F is as follows:

$F = MS_0 + P_1 MS_1 + P_2 MS_2 + \dots$ where

- MS_0 is minimum sum obtained by setting $P_1 = P_2 .. = 0$
- MS_1 is minimum sum obtained by setting $P_1 = 1$, $P_j = 0$ $(j \neq 1)$, and replacing all 1's on the map with don't cares.

Previously, G = A'B' + ACD + EA'D + FAD.

THANK YOU