1EC306 Digital Electronics

AB \ CD	00	01	11	10
00				
01			0	0
11	X	X	X	X
10	0	0	X	X

AB \ CD	00	01	11	10
00	1 0	0 1	0 3	1 2
01	0 4	1 5	1 7	0 6
11	0 12	1 13	1 15	0 14
10	1 8	0 9	0 11	1 10

Guess today's topic???

Karnaugh Map

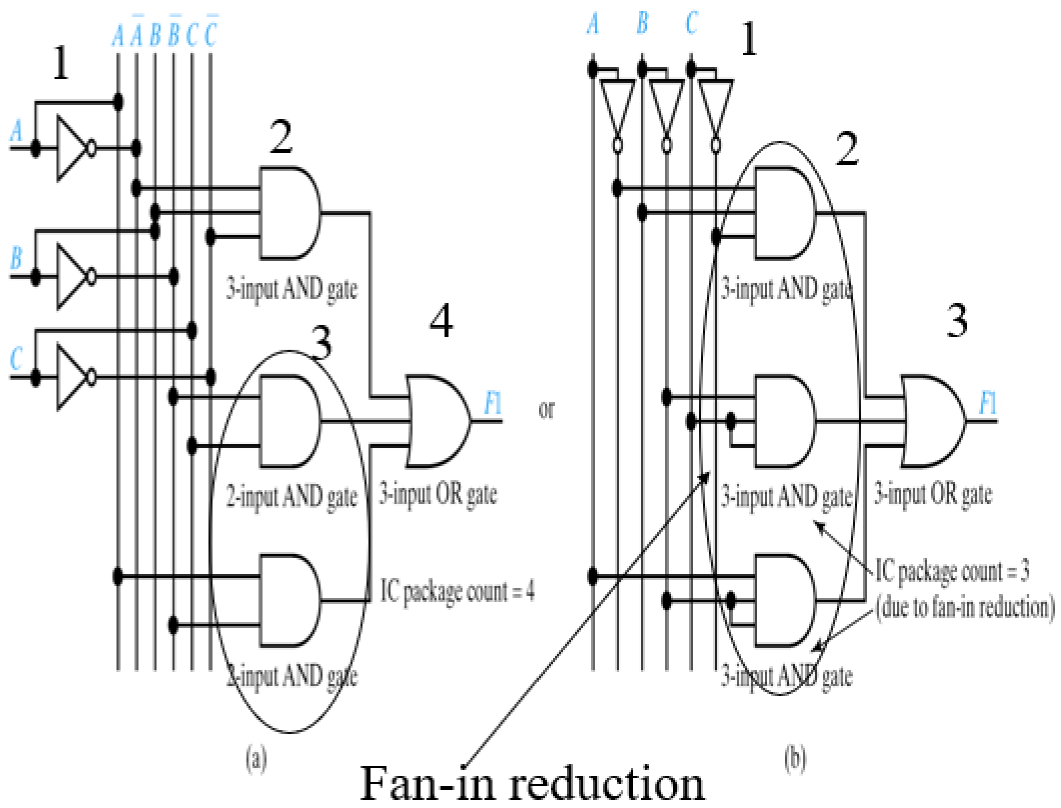
- Algebraic procedures:
  - Difficult to apply in a systematic way.
  - Difficult to tell when you have arrived at a minimum solution.
- Karnaugh map (K-map) can be used to minimize functions of up to 6 variables.
  - K-map is directly applied to two-level networks composed of AND and OR gates.
    - Sum-of-products, (SOP)
    - Product-of-sum, (POS).

Minimum SOP

- It has a minimum no. of terms.
  - That is, it has a minimum number of gates.
- It has a minimum no. of gate inputs.
  - That is, minimum no. of literals.
  - Each term in the minimum SOP is a prime implicant, i.e., it cannot be combined with others.
- It may not be unique.
  - Depend on the order in which terms are combined or eliminated.

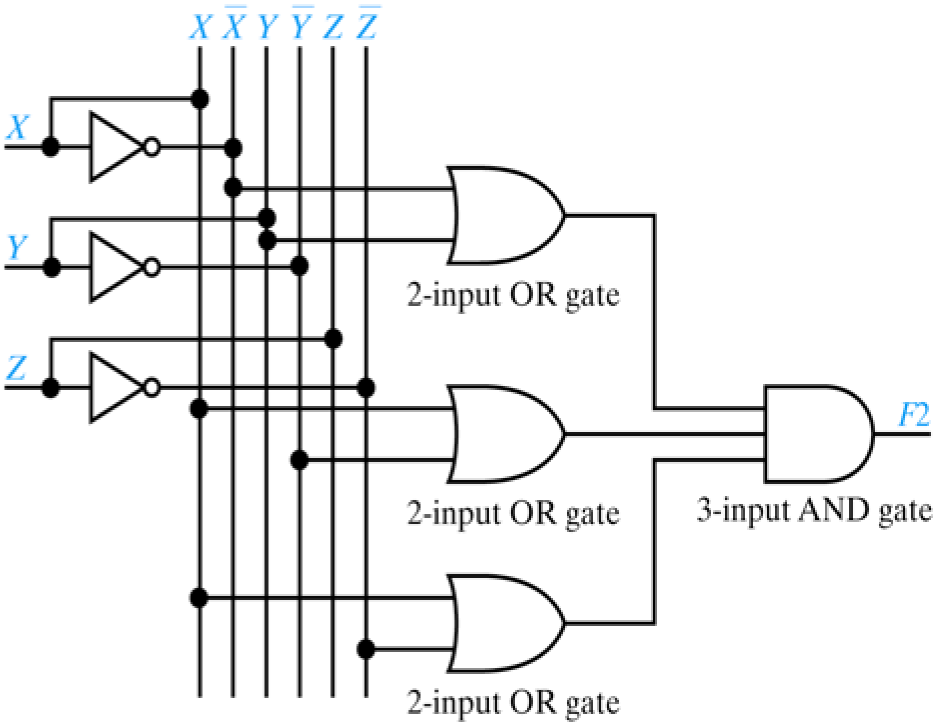
Minimum SOP

- Example: vertical input scheme



Minimum POS

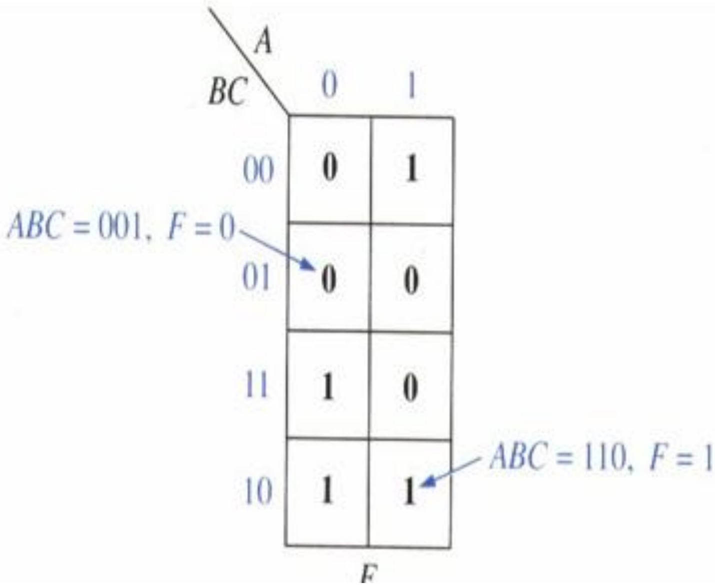
- Example: Vertical input scheme



3 Variable K-Map

- Note BC is listed in the order of 00, 01, 11, 10. (Gray code)
- Minterms in adjacent squares that differ in only one variable can be combined using XY' + XY = X.

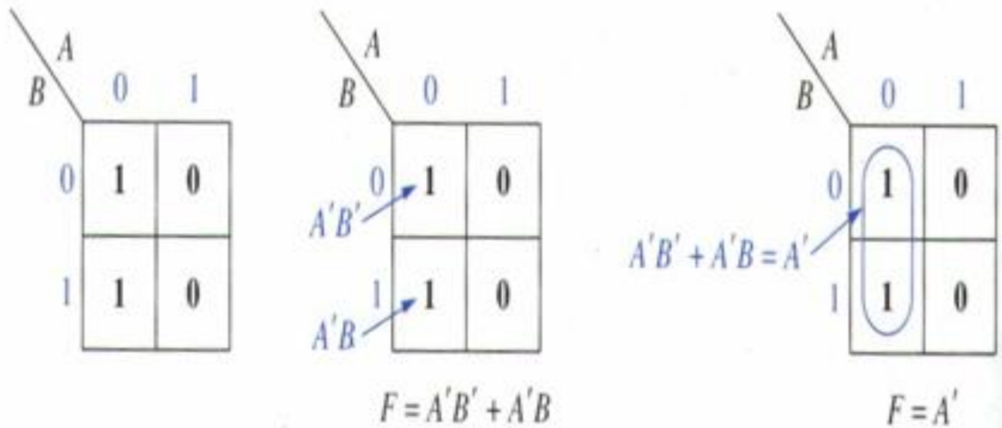
A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0



2 Variable K-Map

- Place 1s and 0s from the truth table in the K-map.
- Each square of 1s = minterms.
- Minterms in adjacent squares can be combined since they differ in only one variable. Use XY' + XY = X.

A B	F
0 0	1
0 1	1
1 0	0
1 1	0



K-Map Example

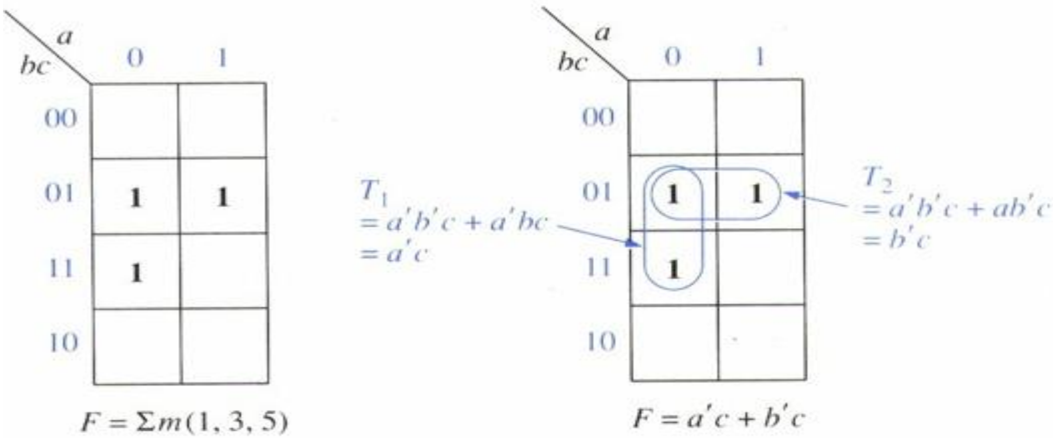
– K-map of $F(a,b,c) = \sum m(1,3,5)$
$= \prod M(0,2,4,6,7)$

bc \ a	0	1
00	0 (0)	0 (4)
01	1 (1)	1 (5)
11	1 (3)	0 (7)
10	0 (2)	0 (6)

Karnaugh Map of
$F(a, b, c) = \sum m(1, 3, 5) = \prod M(0, 2, 4, 6, 7)$

Simplification Example

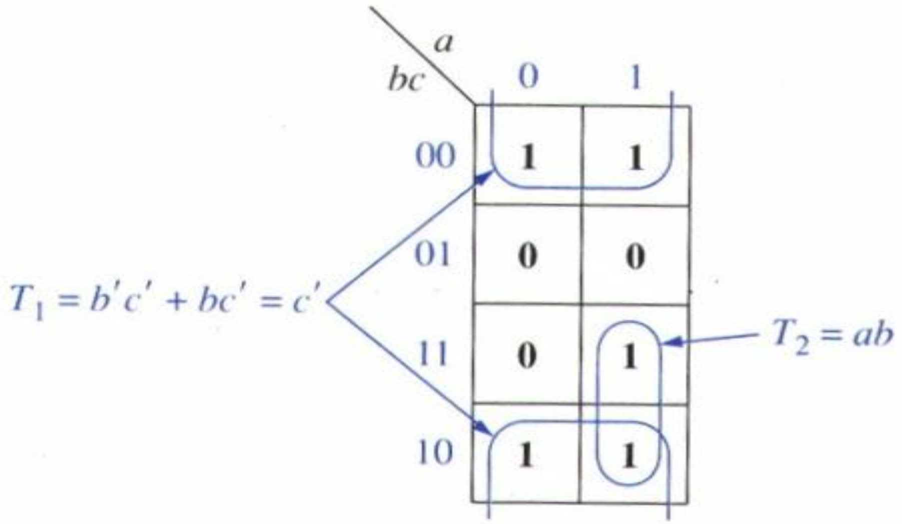
- Exercise. Simplify: $F(a,b,c) = \sum m(1,3,5)$
  - Procedure: place minterms into map.
  - Select adjacent 1's in group of two 1's or four 1's etc.
  - Kick off x and x'.



(a) Plot of minterms (b) Simplified form of F

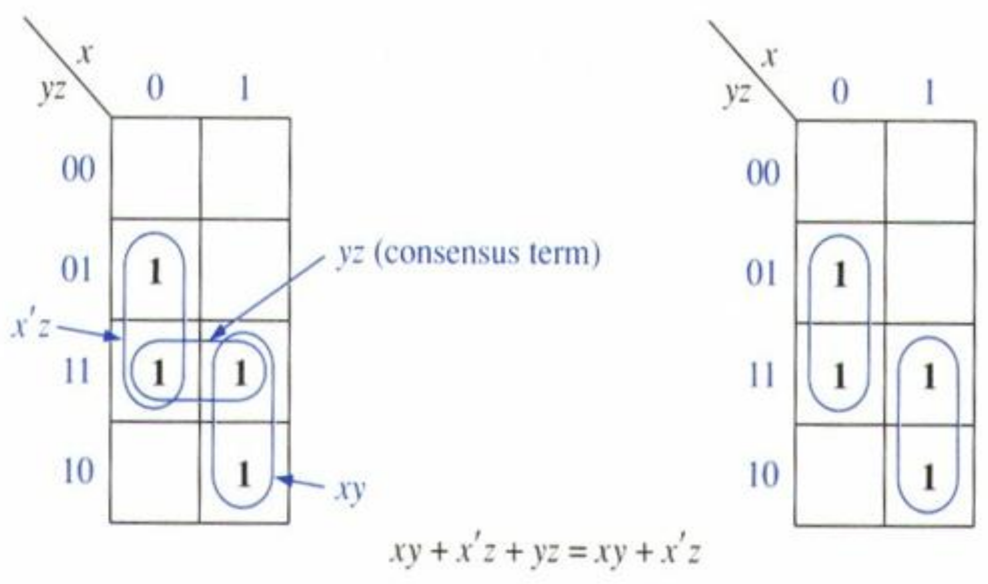
More Example

– The complement of F
  - Using four 1's to eliminate two variables.



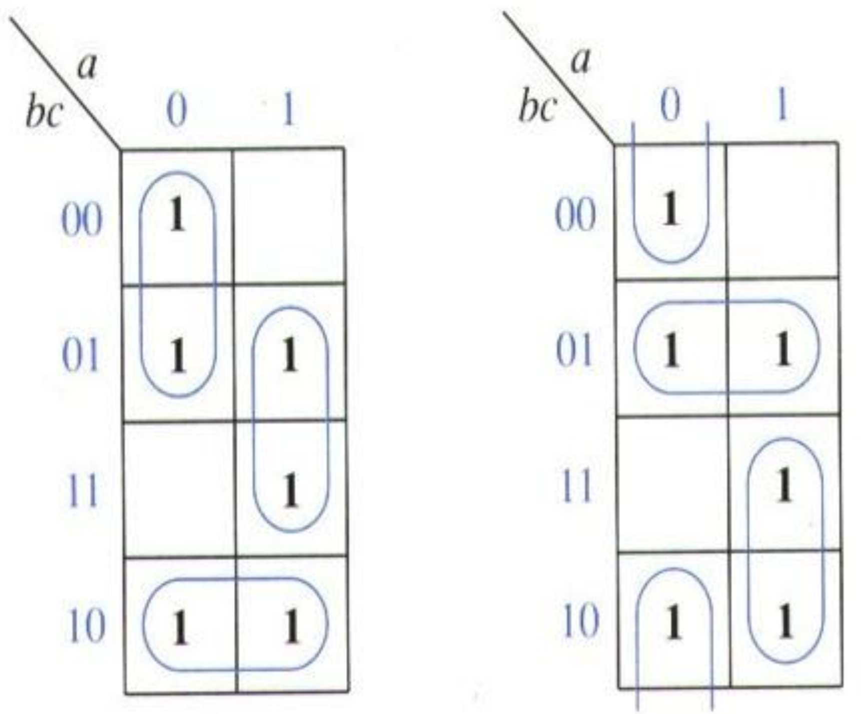
Redundant Term

- If a term is covered by two other terms, that term is redundant. That is, it is a consensus term.
- Example: yz is the redundant.



More than 2 Solution

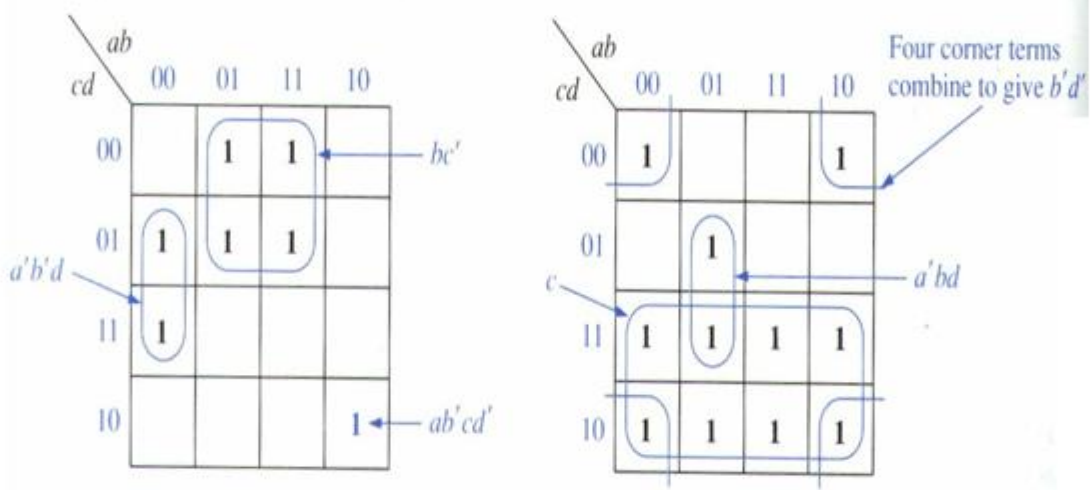
- $F = \sum m(0,1,2,5,6,7)$



$F = a'b' + bc' + ac$

$F = a'c' + b'c + ab$

- Minterms are combined in groups of 2, 4, or 8 to eliminate 1, 2, 3 variables.
- Corner terms.

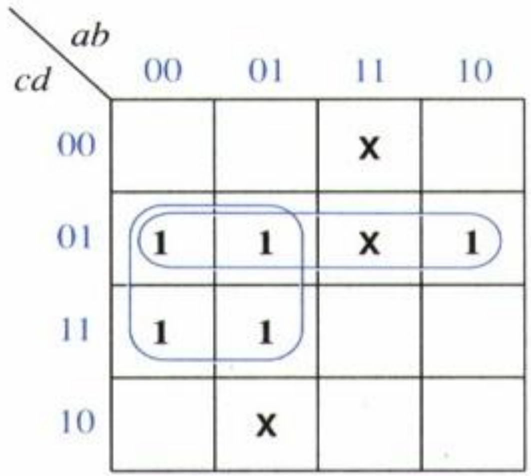


$f_1 = \Sigma m(1, 3, 4, 5, 10, 12, 13)$
$= bc' + a'b'd + ab'cd'$

$f_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$
$= c + b'd' + a'bd$

Simplification with Don’t Care

- Don’t care “x” is covered if it helps. Otherwise leave it along.



$$f = \Sigma m(1, 3, 5, 7, 9) + \Sigma d(6, 12, 13)$$
$$= a'd + c'd$$

Assessment

- Cover 0's to get simplified POS.
  - We want 0 in each term.

$$f = x'z' + wyz + w'y'z' + x'y$$

First, the 1's of f are plotted in Fig. 5–14. Then, from the 0's,

$$f' = y'z + wxz' + w'xy$$

and the minimum product of sums for f is

$$f = (y + z')(w' + x' + z)(w + x' + y')$$

yz \ wx	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	1	0	1	1
10	1	0	0	1


Thank You!