II YEAR/ III SEMESTER

UNIT 1 – **MINIMIZATION TECHNIQUES AND LOGIC GATES**

Learning Objectives

In this chapter you will learn about:

1. Non-positional number system
2. Positional number system
3. Decimal number system
4. Binary number system
5. Octal number system
6. Hexadecimal number system

number systems

Two types of number systemsare:

- Non-positional number systems
- Positional number systems

Non-positional Number Systems

Characteristics

- Use symbols such as I for 1, II for 2, III for 3, IIII
- for 4, IIIII for 5, etc
- Each symbol represents the same value regardless of its position in the number
- The symbols are simply added to find out the value of a particular number

Disadvantages:

It is difficult to perform arithmetic with such a number system

Positional Number Systems

Characteristics

1. Use only a few symbols called digits
2. These symbols represent different values depending on the position they occupy in the number

Decimal Number System

Characteristics

- A positional number system
- Has 10 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). Hence, its base = 10
- The maximum value of a single digit is 9 (one less than the value of the base)
- Each position of a digit represents a specific power of the base (10)
- We use this number system in our day-to-day life

Decimal Number System

Example

$$2586_{10} = (2 \times 10^3) + (5 \times 10^2) + (8 \times 10^1) + (6 \times 10^0)$$

$$= 2000 + 500 + 80 + 6$$

Binary Number System

Characteristics

- A positional number system
- Has only 2 symbols or digits (0 and 1). Hence its base = 2
- The maximum value of a single digit is 1 (one less than the value of the base)
- Each position of a digit represents a specific power of the base (2)
- This number system is used in computers

Binary Number System

Example

$$10101_2 = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) \times (1 \times 2^0)$$

$$= 16 + 0 + 4 + 0 + 1$$

$$= 21_{10}$$

Representing Numbers in Different Number Systems

In order to be specific about which number system we are referring to, it is a common practice to indicate the base as a subscript. Thus, we write:

$$10101_2 = 21_{10}$$

Bit

- Bit stands for **bi**nary digi**t**
- A bit in computer terminology means either a 0 or a 1
- A binary number consisting of *n* bits is called an n-bit number

Octal Number System

Characteristics

- A positional number system
- Has total 8 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7). Hence, its base = 8
- The maximum value of a single digit is 7 (one less than the value of the base
- Each position of a digit represents a specific power of the base (8)

Octal Number System

- Since there are only 8 digits, 3 bits ($2^3 = 8$) are sufficient to represent any octal number in binary

Example

$$2057_8 = (2 \times 8^3) + (0 \times 8^2) + (5 \times 8^1) + (7 \times 8^0)$$

$$= 1024 + 0 + 40 + 7$$

$$= 1071_{10}$$

Hexadecimal Number System

Characteristics

- A positional number system
- Has total 16 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F). Hence its base = 16
- The symbols A, B, C, D, E and F represent the decimal values 10, 11, 12, 13, 14 and 15 respectively
- The maximum value of a single digit is 15 (one less than the value of the base)

Hexadecimal Number System

Each position of a digit represents a specificpower of the base (16)

- Since there are only 16 digits, 4 bits ($2^4 = 16$) are sufficient to represent any hexadecimal number in binary

Example

$$1AF_{16} = (1 \times 16^2) + (A \times 16^1) + (F \times 16^0)$$
$$= 1 \times 256 + 10 \times 16 + 15 \times 1$$
$$= 256 + 160 + 15$$
$$= 431_{10}$$

Converting a Number of Another Base to a Decimal Number

Method

Step 1: Determine the column (positional) value of each digit

Step 2: Multiply the obtained column values by the digits in the corresponding columns

Step 3: Calculate the sum of these products

Converting a Number of Another Base to a Decimal Number

Example

$4706_8 = ?_{10}$

Common values multiplied by the corresponding digits

$$4706_8 = 4 \times 8^3 + 7 \times 8^2 + 0 \times 8^1 + 6 \times 8^0$$
$$= 4 \times 512 + 7 \times 64 + 0 + 6 \times 1$$
$$= 2048 + 448 + 0 + 6$$
$$= 2502_{10}$$

Sum of these products

Converting a Decimal Number to a Number of Another Base

Division-Remainder Method

Step 1: Divide the decimal number to be converted by the value of the new base

Step 2: Record the remainder from Step 1 as the rightmost digit (least significant digit) of the new base number

Step 3: Divide the quotient of the previous divide by the new base

Converting a Decimal Number to a Number of Another Base

Step 4: Record the remainder from Step 3 as the next digit (to the left) of the new base number

Repeat Steps 3 and 4, recording remainders from right to left, until the quotient becomes zero in Step 3

Note that the last remainder thus obtained will be the most significant digit (MSD) of the new base number

Converting a Decimal Number to a Number of Another Base

Example

$952_{10} = ?_8$

Solution:

		Remainders
8	952	
	119	0
	14	7
	1	6
	0	1

Hence, $952_{10} = 1670_8$

Converting a Number of Some Base to a Number of Another Base

Method

Step 1: Convert the original number to a decimal number (base 10)

Step 2: Convert the decimal number so obtained to the new base number

Converting a Number of Some Base to a Number of Another Base

Example

$545_6 = ?_4$

Solution:

Step 1: Convert from base 6 to base 10

$$545_6 = 5 \times 6^2 + 4 \times 6^1 + 5 \times 6^0$$
$$= 5 \times 36 + 4 \times 6 + 5 \times 1$$
$$= 180 + 24 + 5$$
$$= 209_{10}$$

Converting a Number of Some Base to a Number of Another Base

Step 2: Convert 209_{10} to base 4

Divisor	Quotient	Remainders
4	209	
	52	1
	13	0
	3	1
	0	3

Hence, $209_{10} = 3101_4$

So, $545_6 = 209_{10} = 3101_4$

Thus, $545_6 = 3101_4$

Shortcut Method for Converting a Binary Number to its Equivalent Octal Number

Method

Step 1: Divide the digits into groups of threestarting from the right

Step 2: Convert each group of three binary digits to one octal digit using the method of binary to decimal conversion

Shortcut Method for Converting a Binary Number to its Equivalent Octal Number

- Example
  - $1101010_2 = ?_8$
  - Step 1: Divide the binary digits into groups of 3 starting from right
    - <u>001</u> <u>101</u> <u>010</u>
  - Step 2: Convert each group into one octal digit

$$001_2 = 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1$$
$$101_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 5$$
$$010_2 = 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2$$

Hence, $1101010_2 = 152_8$

Shortcut Method for Converting an Octal Number to Its Equivalent Binary Number

Method

Step 1: Convert each octal digit to a 3 digit binary number (the octal digits may be treated as decimal for this conversion)

Step 2: Combine all the resulting binary groups (of 3 digits each) into a single binary number

Shortcut Method for Converting an Octal Number to Its Equivalent Binary Number

Example

$562_8 = ?_2$

Step 1: Convert each octal digit to 3 binary digits

$5_8 = 101_2$, $6_8 = 110_2$, $2_8 = 010_2$

Step 2: Combine the binary groups

$$562_8 = \underset{5}{\underline{101}}\ \underset{6}{\underline{110}}\ \underset{2}{\underline{010}}$$

Hence, $562_8 = 101110010_2$

Shortcut Method for Converting a Binary Number to its Equivalent Hexadecimal Number

Method

Step 1 : Divide the binary digits into groups of four starting from the right

Step 2 : Combine each group of four binary digits to one hexadecimal digit

Shortcut Method for Converting a Binary Number to its Equivalent Hexadecimal Number

Example

$111101_2 = ?_{16}$

Step 1: Divide the binary digits into groups of four starting from the right

0011 1101

Step 2: Convert each group into a hexadecimal digit

$0011_2 = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3_{10} = 3_{16}$

$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 13_{10} = D_{16}$

Hence, $111101_2 = 3D_{16}$

Shortcut Method for Converting a Hexadecimal Number to its Equivalent Binary Number

Method

Step 1: Convert the decimal equivalent of each hexadecimal digit to a 4 digit binary number

Step 2: Combine all the resulting binary groups (of 4 digits each) in a single binary number

Shortcut Method for Converting a Hexadecimal Number to its Equivalent Binary Number

Example

$2AB_{16} = ?_2$

Step 1: Convert each hexadecimal digit to a 4 digit binary number

$2_{16} = 2_{10} = 0010_2$
$A_{16} = 10_{10} = 1010_2$
$B_{16} = 11_{10} = 1011_2$

Shortcut Method for Converting a Hexadecimal Number to its Equivalent Binary Number

Step 2: Combine the binary groups

$$2AB_{16} = \underset{2}{\underline{0010}} \quad \underset{A}{\underline{1010}} \quad \underset{B}{\underline{1011}}$$

Hence, $2AB_{16} = 001010101011_2$

Assessment

1.What is Number system?

2.List the different types of number systems.

3.How will you convert the binary number to hexadecimal number system?