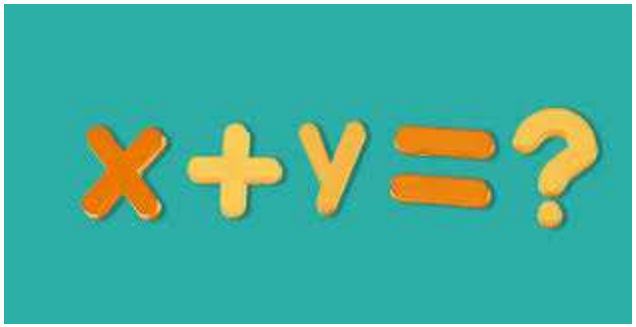
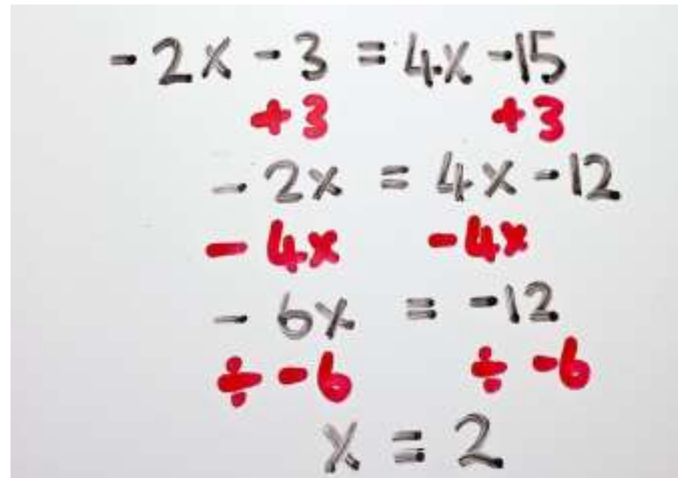
SNS COLLEGE OF ENGINEERING

(Autonomous)

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

19EC306 Digital Electronics





Guess today's topic???

Boolean algebra

Boolean Algebra

When we learned numbers like 1, 2, 3, we also then learned how to add, multiply, etc. with them. Boolean Algebra covers operations that we can do with 0's and 1's. Computers do these operations ALL THE TIME and they are basic building blocks of computation inside your computer program.

Axioms, laws, theorems

We need to know some rules about how those 0's and 1's can be operated on together. There are similar axioms to decimal number algebra, and there are some laws and theorems that are good for us to use to simplify our operation.

Boolean Algebra

A Boolean algebra comprises...

A set of elements B

Binary operators {+ , •} Boolean sum and product

A unary operation { ' } (or { }) example: A’ or A

…and the following axioms

1. The set B contains at least two elements {a b} with $a \neq b$
2. Closure: a+b is in B a•b is in B
3. Commutative: a+b = b+a a•b = b•a
4. Associative: a+(b+c) = (a+b)+c a•(b•c) = (a•b)•c
5. Identity: a+0 = a a•1 = a
6. Distributive: a+(b•c)=(a+b)•(a+c) a•(b+c)=(a•b)+(a•c)
7. Complementarity: a+a' = 1 a•a' = 0

Digital (binary) logic is a Boolean algebra

Substitute

{0, 1} for B

AND for • Boolean Product.

OR for + Boolean Sum.

NOT for ‘ Complement.

All the axioms hold for binary logic

Definitions

Boolean function

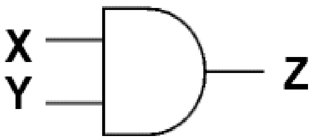
- Maps inputs from the set {0,1} to the set {0,1}

Boolean expression

- An algebraic statement of Boolean variables and operators

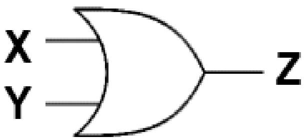
Logic Gates (AND, OR, Not) & Truth Table

- AND X•Y XY



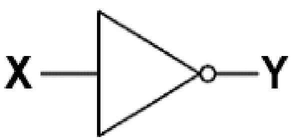
X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

- OR X+Y



X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

- NOT $\overline{X}$ X'



X	Y
0	1
1	0

Logic functions and Boolean algebra

- *Any* logic function that is expressible as a truth table can be written in Boolean algebra using +, •, and '

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

Z=X•Y

X	Y	X'	Z
0	0	1	0
0	1	1	1
1	0	0	0
1	1	0	0

Z=X'•Y

X	Y	X'	Y'	X • Y	X' •Y'	Z
0	0	1	1	0	1	1
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	1	0	0	1	0	1

Z=(X•Y)+(X' •Y')

Two key concepts

- Duality
  All Boolean expressions have logical duals
  Any theorem that can be proved is also proved for its dual
  Replace: • with +, + with •, 0 with 1, and 1 with 0
  Leave the variables unchanged

- De Morgan's Theorem
  Procedure for complementing Boolean functions
  Replace: • with +, + with •, 0 with 1, and 1 with 0
  Replace all variables with their complements

Useful laws and theorems

Identity:	$X + 0 = X$	Dual: $X \bullet 1 = X$
Null:	$X + 1 = 1$	Dual: $X \bullet 0 = 0$
Idempotent:	$X + X = X$	Dual: $X \bullet X = X$
Involution:	$(X')' = X$	
Complementarity:	$X + X' = 1$	Dual: $X \bullet X' = 0$
Commutative:	$X + Y = Y + X$	Dual: $X \bullet Y = Y \bullet X$
Associative:	$(X+Y)+Z=X+(Y+Z)$	Dual: $(X \bullet Y) \bullet Z=X \bullet (Y \bullet Z)$
Distributive:	$X \bullet (Y+Z)=(X \bullet Y)+(X \bullet Z)$	Dual: $X+(Y \bullet Z)=(X+Y) \bullet (X+Z)$
Uniting:	$X \bullet Y+X \bullet Y'=X$	Dual: $(X+Y) \bullet (X+Y')=X$

Assessment

- Example 1: Prove the uniting theorem-- $X \bullet Y + X \bullet Y' = X$

Distributive	$X \bullet Y + X \bullet Y' = X \bullet (Y + Y')$
Complementarity	$= X \bullet (1)$
Identity	$= X$

- Example 2: Prove the absorption theorem-- $X + X \bullet Y = X$

Identity	$X + X \bullet Y = (X \bullet 1) + (X \bullet Y)$
Distributive	$= X \bullet (1 + Y)$
Null	$= X \bullet (1)$
Identity	$= X$

Assessment

- Example: $F=(A+B)\bullet(A'+C)$, so $F'=(A'\bullet B')+(A\bullet C')$

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

A	B	C	F′
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0


Thank You!