TOPIC:11.- Proof methods and strategy.

The Theory of Inference for Predicate calculus

Universal Specification (US)

If a statement of the form $(\forall x)[A(x)]$ is assumed to be true, then the universal quantifier can be dropped to obtain $A(y)$ is true for any arbitrary object 'y' in the universe.

Existential Specification (ES)

From $\exists x(A(x))$ one can conclude $A(y)$, provided that y is not free in any given premise and also not free in any prior step of the derivation.

Universal Generalization (UG)

From $A(y)$ one can conclude

$$A(y) \Rightarrow (\forall x)(A(x))$$

Existential Generalization (EG)

$$A(y) \Rightarrow (\exists x)(A(x))$$

1. Express the statement "Every student in this class has completed Assignment - I" as a quantifiers.

Let $C(x)$: x is in this class

$A(x)$: x has completed Assignment - I

For all x, if x is in this class, then x has completed Assignment - I.

∴ Its symbolic form $(\forall x)\,(C(x) \rightarrow A(x))$

2.

Write each of the following in symbolic form.

All men are good (b) No men are good

Some men are good (e) Some men are not good

Let $M(x)$: x is a man

$G(x)$: x is good.

i) For all x, x is a man, then x is good.

∴ $(\forall x)\,(M(x) \rightarrow G(x))$

(b) For all x, if x is a man, then x is not good

$\therefore \quad (\forall x)\,(M(x) \rightarrow \neg G(x))$

(c) There exists an x, x is a man and x is g

$\therefore \quad (\exists x)\,(M(x) \wedge G(x))$

(d) There exists an x, x is a man and x is not good.

$(\exists x)\,(M(x) \wedge \neg G(x))$

3.

Show that $(\forall x)(P(x) \rightarrow Q(x)) \wedge (\forall x)(Q(x) \rightarrow R$

$\Rightarrow (\forall x)(P(x) \rightarrow R(x))$.

$\{1\}$	1) $(\forall x)\,(P(x) \rightarrow Q(x))$	Rule P
$\{1\}$	2) $P(y) \rightarrow Q(y)$	Rule US
$\{3\}$	3) $(\forall x)(Q(x) \rightarrow R(x))$	Rule P
$\{3\}$	4) $Q(y) \rightarrow R(y)$	Rule US
$\{1,3\}$	5) $P(y) \rightarrow R(y)$	Rule T $(P \rightarrow Q,\, Q \rightarrow R \Rightarrow P \rightarrow R)$
$\{1,3\}$	6) $(\forall x)\,(P(x) \rightarrow R(x))$	Rule UG

4)

, Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x) P(x) \vee (\exists x) Q(x)$

We use indirect method, by assuming

$\neg[(\forall x) P(x) \vee (\exists x)(Q(x))]$ as an additional premise

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{1}	1) $\neg[(\forall x) P(x) \vee (\exists x) Q(x)]$	Rule P
{1}	2) $(\exists x) \neg P(x) \wedge (\forall x) \neg Q(x)$	Rule T (Demorgan's)
{1}	3) $(\exists x) \neg P(x)$	Rule T $(P \wedge Q \Rightarrow P)$
{1}	4) $(\forall x) \neg Q(x)$	Rule T $(P \wedge Q \Rightarrow Q)$
{1}	5) $\neg P(y)$	Rule ES
{1}	6) $\neg Q(y)$	Rule US
{1}	7) $\neg P(y) \wedge \neg Q(y)$	Rule T $(P, Q \Rightarrow P \wedge Q$
{1}	8) $\neg (P(y) \vee Q(y))$	Rule T (Demorgan's)
{9}	9) $(\forall x)(P(x) \vee Q(x))$	Rule P

		Rule US
{9}	10) $P(y) \vee Q(y)$	
{1,9}	11) $[P(y) \vee Q(y)] \wedge \neg [P(y) \vee Q(y)]$	Rule T $(P, Q \Rightarrow P \wedge Q)$
{1,9}	12) F	

5) Verify the validity of the following argument. Every living thing is a plant or animal. John's gold fish is alive and it is not a plant. All animals have hearts. Therefore John's gold fish has a heart.

Let $L(x)$: x is a living thing

$P(x)$: x is a plant

$A(x)$: x is an animal

$H(x)$: x has a heart

Then, the given premises are

(1) $(\forall x)\,[L(x) \rightarrow (P(x) \vee A(x))]$

(2) $L(j) \wedge \neg P(j)$

(3) $(\forall x)\,[A(x) \rightarrow H(x)]$

Conclusion is $H(j)$

{1}	1) $(\forall x)\left[L(x) \rightarrow (P(x) \vee A(x))\right]$	Rule P
{1}	2) $L(j) \rightarrow P(j) \vee A(j)$	Rule US
{3}	3) $L(j) \wedge \neg P(j)$	Rule P
{3}	4) $L(j)\ ,\ \neg P(j)$	Rule T $(P \wedge Q \Rightarrow P, Q)$
{1, 3}	5) $P(j) \vee A(j)$	Rule T $(P, P \rightarrow Q \Rightarrow Q)$
{1, 3}	6) $\neg P(j) \rightarrow A(j)$	Rule T $(P \rightarrow Q \Rightarrow \neg P \vee Q)$
{7}	7) $(\forall x)\left(A(x) \rightarrow H(x)\right)$	Rule P
{7}	8) $A(j) \rightarrow H(j)$	Rule US
{1,3,7}	9) $\neg P(j) \rightarrow H(j)$	Rule T $(P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R)$
{1,3,7}	10) $H(j)$	Rule T $(P, P \rightarrow Q \Rightarrow Q)$