



① Demonstrate that R is a valid inference from the premises  $P \rightarrow Q$ ,  $Q \rightarrow R$  and P.

Here given premises are

- (1)  $P \rightarrow Q$
- (2)  $Q \rightarrow R$
- (3) P

$\{1\}$	1) $P \rightarrow Q$	Rule P
$\{2\}$	2) P	Rule P
$\{1,2\}$	3) Q	Rule T ( $P, P \rightarrow Q \Rightarrow Q$ )
$\{4\}$	4) $Q \rightarrow R$	Rule P
$\{1,2,4\}$	5) R	Rule T ( $P, P \rightarrow Q \Rightarrow Q$ )



2. show that  $(P \rightarrow Q) \wedge (R \rightarrow S), (Q \rightarrow M) \wedge (S \rightarrow N),$

$\neg(M \wedge N)$  and  $(P \rightarrow R) \Rightarrow \neg P.$

Given premises are  $(P \rightarrow Q) \wedge (R \rightarrow S),$   
 $(Q \rightarrow M) \wedge (S \rightarrow N), \neg(M \wedge N)$  and  $(P \rightarrow R)$

Conclusion is  $\neg P.$

{1}	1) $(P \rightarrow Q) \wedge (R \rightarrow S)$	Rule P
{1}	2) $P \rightarrow Q$	Rule T ( $P \wedge Q \Rightarrow P$ )
{1}	3) $R \rightarrow S$	Rule T ( $P \wedge Q \Rightarrow Q$ )
{4}	4) $(Q \rightarrow M) \wedge (S \rightarrow N)$	Rule P
{4}	5) $Q \rightarrow M$	Rule T ( $P \wedge Q \Rightarrow P$ )
{4}	6) $S \rightarrow N$	Rule T ( $P \wedge Q \Rightarrow Q$ )
{1,4}	7) $P \rightarrow M$	Rule T ( $P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$ )
{1,4}	8) $R \rightarrow N$	Rule T ( $P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$ )
{9}	9) $P \rightarrow R$	Rule P



{1,4,9}	10) $P \rightarrow N$	Rule T ( $P \rightarrow a, a \rightarrow R \Rightarrow P \rightarrow R$ )
{1,4,9}	11) $\neg N \rightarrow \neg P$	Rule T (Taking $\neg$ )
{1,4}	12) $\neg M \rightarrow \neg P$	Rule T (Taking $\neg$ )
{1,4,9}	13) $(\neg M \vee \neg N) \rightarrow \neg P$	Rule T ( $a \rightarrow b, c \rightarrow b \Rightarrow (a \vee c) \rightarrow b$ )
{1,4,9}	14) $\neg (M \wedge N) \rightarrow \neg P$	Rule T (Demorgan's law)
{15}	15) $\neg (M \wedge N)$	Rule P
{1,4,9,15}	16) $\neg P$	Rule T ( $P, P \rightarrow Q \Rightarrow Q$ )

3.

Prove that the following argument is valid:

$$p \rightarrow \neg q, r \rightarrow q, r \Rightarrow \neg p$$

Given premises are  $p \rightarrow \neg q, r \rightarrow q, r$

Conclusion is  $\neg p$ .



$\{1\}$	1) $\gamma$	Rule P
$\{2\}$	2) $\gamma \rightarrow q$	Rule P
$\{1,2\}$	3) $q$	Rule T ( $P, P \rightarrow q \Rightarrow q$ )
$\{4\}$	4) $p \rightarrow \neg q$	Rule P
$\{1,2,4\}$	5) $\neg p$	Rule T ( $P \rightarrow \neg q, q \Rightarrow \neg P$ )