Logistic and production Models

i) Supply chain optimization

In a broad sense, a supply chain may be defined as a network of connected and interdependent organizational units that operate in a coordinated way to manage, control and improve the flow of materials and information originating from the suppliers and reaching the end customers, after going through the procurement, processing and distribution subsystems of a company. The aim of the integrated planning and operations of the supply chain is to combine and evaluate from a systemic perspective the decisions made and the actions undertaken within the various sub processes that compose the logistic system of a company.

Many manufacturing companies, such as those operating in the consumer goods industry, have concentrated their efforts on the integrated operations of the supply chain, even to the point of incorporating parts of the logistic chain that are outside the company, both upstream and downstream. The major purpose of an integrated logistic process is to minimize a function expressing the total cost, which comprises processing costs, transportation costs for procurement and distribution, inventory costs and equipment costs. Note that

![Diagram of a global supply chain](image)

*Figure 14.1  An example of global supply chain*

the optimization of the costs for each single phase does not generally imply that the minimum total cost of the entire logistic process has been achieved, so that a holistic
perspective is required to attain a really optimized supply chain. The need to optimize the logistic chain, and therefore to have models and computerized tools for medium-term planning and for capacity analysis, is particularly critical in the face of the high complexity of current logistic systems, which operate in a dynamic and truly competitive environment. We are referring here to manufacturing companies that produce a vast array of products and that usually rely on a multicentric logistic system, distributed over several plants and markets, characterized by large investments in highly automated technology, by an intensive usage of the available production capacity and by short-order processing cycles. The features of the logistic system we have described reflect the profile of many enterprises operating in the consumer goods industry. In the perspective outlined above, the aim of a medium-term planning process is therefore to devise an optimal logistic production plan, that is, a plan that is able to minimize the total cost, understood as the sum of procurement, processing, storage, distribution costs and the penalty costs associated with the failure to achieve the predefined service level. However, to be implemented in practice, an optimal logistic production plan should also be feasible, that is, it should be able to meet the physical and logical constraints imposed by limits on the available production capacity, specific technological conditions, the structure of the bill of materials, the configuration of the logistic network, minimum production lots, as well as any other condition imposed by the decision makers in charge of the planning process.

Optimization models represent a powerful and versatile conceptual paradigm for analyzing and solving problems arising within integrated supply chain planning, and for developing the necessary software. Due to the complex interactions occurring between the different components of a logistic production system, other methods and tools intended to support the planning activity seem today inadequate, such as electronic spreadsheets, simulation systems and planning modules at infinite capacity included in enterprise resource planning software. Conversely, optimization models enable the development of realistic mathematical representations of a logistic production system, able to describe with reasonable accuracy the complex relationships among critical components of the logistic system, such as capacity, resources, plans, inventory, batch sizes, lead times and logistic flows, taking into account the various costs. Moreover, the evolution of information technologies and the latest developments in optimization algorithms mean that decision support systems based on optimization models for logistics planning can be efficiently developed.

ii) Optimization models for logistics planning
In this section some optimization models that may be used to represent the most relevant features of logistic production systems are described. As already observed when introducing sales force planning models in Chapter 13, for the sake of simplicity we have chosen to illustrate for each model a single feature of a logistic system. Readers should keep in mind that real-world logistic production systems feature simultaneously more than one of the elements considered, so that the models developed in applications, such as the business case studies presented in Section 14.4, will be substantially more complex as they result from the combination of the different features.

Before proceeding with the description of specific models, it is useful to introduce some notation common to most models presented in this section. The logistic system includes $I$ products, which will be denoted by the index $i \in I = \{1, 2, \ldots, I\}$. The planning horizon is subdivided into $T$ time intervals $t \in T = \{1, 2, \ldots, T\}$, generally of equal length and usually corresponding to weeks or months.

The manufacturing process has at its disposal a set of critical resources shared among the different products and available in limited quantities. These resources may consist of production and assembly lines, to manpower, to specific fixtures and tools required by manufacturing. The $R$ critical resources considered in the logistic production system will be denoted by the index $r \in R = \{1, 2, \ldots, R\}$. Whenever a single resource is relevant to the manufacturing process, the index $r$ will be omitted for sake of simplicity.

a) Tactical planning

In its simplest form, the aim of tactical planning is to determine the production volumes for each product over the $T$ periods included in the medium-term planning horizon in such a way as to satisfy the given demand and capacity limits for a single resource, and also to minimize the total cost, defined as the sum of manufacturing production costs and inventory costs.

Therefore consider the decision variables,
- $P_{it} =$ units of product $i$ to be manufactured in period $t$,
- $I_{it} =$ units of product $i$ in inventory at the end of period $t$,
and the parameters
- $d_{it} =$ demand for product $i$ in period $t$,
- $c_{it} =$ unit manufacturing cost for product $i$ in period $t$,
- $h_{it} =$ unit inventory cost for product $i$ in period $t$,
- $e_i =$ capacity absorption to manufacture a unit of product $i$,
$bt =$ capacity available in period $t$.

The resulting optimization problem is formulated as follows:

\[
\begin{align*}
\min & \quad \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} (c_{it} P_{it} + h_{it} I_{it}) \\
\text{s.t.} & \quad P_{it} + I_{i,t-1} - I_{it} = d_{it}, \quad i \in \mathcal{I}, t \in \mathcal{T}, \\
& \quad \sum_{i \in \mathcal{I}} e_i P_{it} \leq b_t, \quad t \in \mathcal{T}, \\
& \quad P_{it}, I_{it} \geq 0, \quad i \in \mathcal{I}, t \in \mathcal{T}.
\end{align*}
\] (14.1) (14.2) (14.3) (14.4)

Constraints (14.2) express the balance conditions among production, inventory and demand, by establishing a connection between successive periods along the planning horizon. Inequalities (14.3) constrain the absorbed capacity not to exceed the available capacity for each period.

Model (14.1) is linear optimization problems which can be therefore solved efficiently even with a very large number of variables and constraints, of the order of a few million, by means of current state-of-art algorithms and computer technologies.

b) **Extra capacity**

A first extension of the basic model (14.1) deals with the possibility of resorting to *extra capacity*, perhaps in the form of overtime, part-time or third-party capacity. In addition to the decision variables already included in model (14.1), we define the variables

\[ Ot = \text{extra capacity used in period } t, \text{ and the parameters} \]

\[ qt = \text{unit cost of extra capacity in period } t. \]

The optimization problem now becomes:

\[
\begin{align*}
\min & \quad \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} (c_{it} P_{it} + h_{it} I_{it}) + \sum_{t \in \mathcal{T}} q_t O_t \\
\text{s.to} & \quad P_{it} + I_{i,t-1} - I_{it} = d_{it}, \quad i \in \mathcal{I}, t \in \mathcal{T}, \\
& \quad \sum_{i \in \mathcal{I}} e_i P_{it} \leq b_t + O_t, \quad t \in \mathcal{T}, \\
& \quad P_{it}, I_{it}, O_t \geq 0, \quad i \in \mathcal{I}, t \in \mathcal{T}.
\end{align*}
\] (14.5) (14.6) (14.7) (14.8)
Constraints (14.7) have been modified to include the available extra capacity. The extended model (14.5) is still a linear optimization problem which can be therefore solved efficiently.

c) Multiple resources

If the manufacturing system requires \( R \) critical resources, a further extension of model (14.1) can be devised by considering multiple capacity constraints. The decision variables already included in model (14.1) remain unchanged, though it is necessary to consider the additional parameters

\[
br_t = \text{quantity of resource } r \text{ available in period } t,
\]

\[
eir = \text{quantity of resource } r \text{ absorbed to manufacture one unit of product } i.
\]

The resulting optimization problem is given by

\[
\begin{align*}
\min & \sum_{i \in I} \sum_{t \in T} (c_{it} P_{it} + h_{it} I_{it}) \\
\text{s.to} & \quad P_{it} + I_{i,t-1} - I_{it} = d_{it}, \quad i \in I, t \in T, \quad (14.10) \\
& \quad \sum_{i \in I} e_{ir} P_{it} \leq b_{rt}, \quad r \in R, t \in T, \quad (14.11) \\
& \quad P_{it}, I_{it} \geq 0, \quad i \in I, t \in T. \quad (14.12)
\end{align*}
\]

Constraints (14.11) have been modified to take into account the upper limits on the capacity of the \( R \) resources in the system. Model (14.9) remains a linear optimization problem which can be solved efficiently.

d.) Backlogging

Another feature that needs to be modeled in some logistic systems is **backlogging**. The term **backlog** refers to the possibility that a portion of the demand due in a given period may be satisfied in a subsequent period, incurring an additional penalty cost. Backlogs are a feature of production systems more likely to occur in B2B or make-to-order manufacturing contexts. In B2C industries, such as mass production consumer goods, on the other hand, one is more likely to find a variant of the backlog, known as **lost sales**, in which unfulfilled demand in a period cannot be transferred to a subsequent
period and is lost. To model backlogging, it is necessary to introduce new decision variables

\[ \text{Bit}_i = \text{units of demand for product } i \text{ delayed in period } t, \text{ and the parameters} \]

\[ g_{it} = \text{unit cost of delaying the demand for product } i \text{ in period } t. \]

The resulting optimization problem is

\[
\min \sum_{i \in \mathcal{I}} \sum_{t \in T} (c_{it} P_{it} + h_{it} l_{it} + g_{it} B_{it}) \tag{14.13}
\]

s.t.

\[ P_{it} + I_{i,t-1} - I_{it} + B_{it} - B_{i,t-1} = d_{it}, \quad i \in \mathcal{I}, t \in T, \tag{14.14} \]

\[ \sum_{i \in \mathcal{I}} c_i P_{it} \leq b_t, \quad t \in T, \tag{14.15} \]

\[ P_{it}, I_{it}, B_{it} \geq 0, \quad i \in \mathcal{I}, t \in T. \tag{14.16} \]

e.) Minimum lots and fixed costs

A further feature often appearing in manufacturing systems is represented by minimum lot conditions: for technical or scale economy reasons, it is sometimes necessary that the production volume for one or more products be either equal to 0 (i.e. the product is not manufactured in a specific period) or not less than a given threshold value, the minimum lot. To incorporate minimum lot conditions into the model, we define the binary decision variables

\[ Y_{it} = \begin{cases} 
1 & \text{if } P_{it} > 0, \\
0 & \text{otherwise},
\end{cases} \]

and the parameters \( l_i = \text{minimum lot for product } i, \)

\[ \gamma = \text{constant value larger than any producible volume for } i. \]

f. Bill of materials

A further extension of the basic planning model deals with the representation of products with a complex structure, described via the so-called bill of materials, where end-items are made by components that in turn may include other components.
Formally, the following parameters are defined to describe the structure of the bill of materials:

\[ a_{ij} = \text{units of product } i \text{ directly required by one unit of product } j, \]

where the term product refers here to both end-items and components at various levels of the bill of materials. For each product \( i \) we assign an external demand \( d_{\text{it}} \) and an internal demand, the latter induced by the requirements of product \( i \) needed to manufacture the components or the end-items for which \( i \) represents a direct component. The external demand for components may originate from other plants of the same manufacturing company or from outside customers that also buy components.

The resulting optimization problem is formulated as

\[
\begin{align*}
\min & \sum_{i \in I} \sum_{t \in T} (c_{\text{it}} P_{\text{it}} + h_{\text{it}} I_{\text{it}}) \\
\text{s.t.} & \quad P_{\text{it}} + I_{\text{i},t-1} - I_{\text{it}} = d_{\text{it}} + \sum_{j \in I, j \neq i} a_{ij} P_{\text{jt}}, \quad i \in I, t \in T, \\
& \quad \sum_{i \in I} e_i P_{\text{it}} \leq b_t, \quad t \in T, \\
& \quad P_{\text{it}}, I_{\text{it}} \geq 0, \quad i \in I, t \in T.
\end{align*}
\]

The balance constraints (14.37) have been modified to take into account the demand internally generated. Model (14.36) is a linear optimization problem which can be therefore solved efficiently.